Indirect Quantum Approximate Optimization Algorithms: application to the TSP

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1 Introduction

We propose an Indirect Quantum Approximate Optimization Algorithm (referred to as IQAOA) where the Quantum Alternating Operator Ansatz takes into consideration a general parameterized family of unitary operators to efficiently model the Hamiltonian that described the set of string vectors. This algorithm creates an efficient alternative to QAOA, where: 1) a Quantum parametrized circuit executed on a quantum machine models the set of string vectors; 2) a Classical meta-optimization loop executed on a classical machine; 3) an estimation of the average cost of each string vector computing, using a well know algorithm coming from the OR community that is problem dependant.

The main advantage is to obtain a quantum circuit with a strongly limited number of gates that could be executed on the noisy current quantum machines. The numerical expriments achieved with IQAOA permits to solve 8 and 9 customer instances TSP using the IBM simulator which are to the best of our knowledge the largest TSP ever solved using a QAOA based approach.

2 IQAOA

2.1 Introduction

Recently, Farhi et al. (2015) [1] introduced a new class of algorithms centered on the alternation between two distinct sets of operators: Hamiltonian and mixing Hamiltonian. This alternation process gives rise to Quantum Approximate Optimization Algorithms, commonly referred to as QAOA. These algorithms represent a hybrid approach in which the classical computer is tasked with exploring the search space to optimize a set of parameters, while the evaluation of probability distributions is executed by a quantum device. It's noteworthy that QAOA does not support local search considerations and offers a comprehensive exploration of the entire search space. This pioneering work was further expanded upon in the renowned publication by Hadfield in 2018 [2]. In their work, they introduced new ansatz that specifically facilitate the exploration of the feasible subspace, ensuring that hard constraints are inherently satisfied. This approach bears a striking resemblance to classical methodologies within the

Operations Research (OR) community, as it involves a meticulous definition of classical operations such as qubit permutations within the qubit-string used for solution modeling.

2.2 Indirect mapping

For a set of n customers, we have $n!$ permutations numbered from 0 to $n!-1$. Let us consider $x \in [0;\, n!-1]$ that models a rank in the list of permutations. To any rank $x \in [0;\, n!-1]$, it is possible to define a subexceedant function f (i.e. $f_i \leq i$) composed of a decomposition in the factorial basis and by consequence the permutation σ. To conclude for any any rank $x \in [0;\, n!-1]$:

- we can compute the subexceedant function by decomposing x in the factorial base;
- we can compute the permutation $\sigma_{f(x)}$ associated to the subexceedant function $f(x)$ [3];
- we can compute the cost of any permutation $\sigma_{f(x)}$ considering: cost $= \sum_{i=0}^{n-2} d_{\sigma_i,\sigma_{i+1}} + d_{\sigma_{n-1},\sigma_0}$ assuming $d_{i,j}$ is the distance from customer i to customer j.

This allows us to define the mapping function that associates a permutation with each rank x. Cheng et al. (1996) [4] pointed out that the most interesting mapping functions are the $1-\text{to}-1$ functions, as they correspond to bijections between the two sets (the set of indirect encoding and the set of solutions). Note that the mapping function just defined is indeed a $1-\text{to}-1$ function, unlike the functions commonly used in Operations Research, which are of the $n-\text{to}-1$ type (including for example the Split method in the VRP that is clearly of the $n-\text{to}-1$ type).

Using such a representation we define IQAOA that extend the classical QAOA approach using a representation of the solution rank only (Fig. 1).

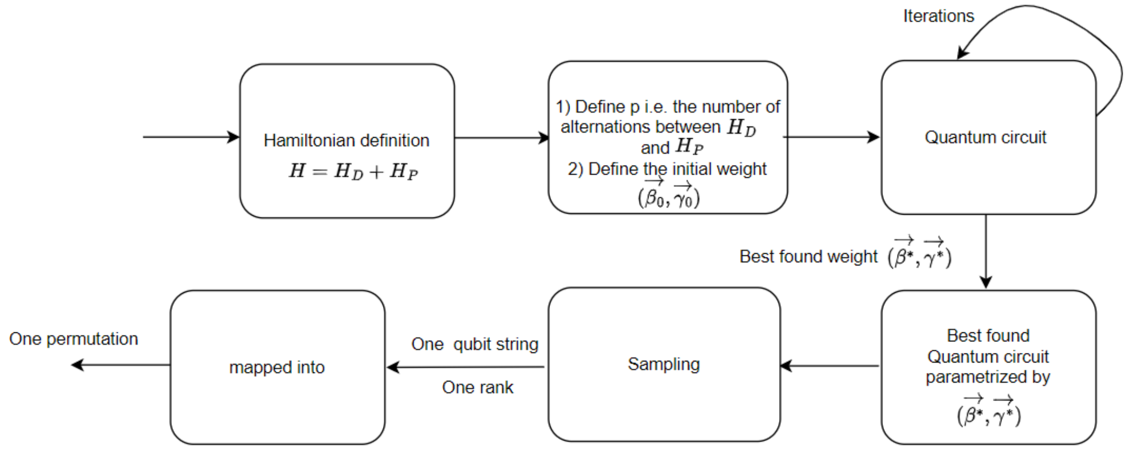

FIG. 1 – IQAOA Principles

2.3 Numerical experiments

IQAOA has been benchmarked on solving TSP from 6 to 9 customers that outperforms all previous attempts that are limited to 4 customers only.

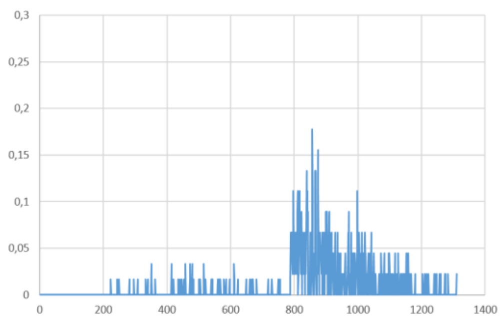

FIG. 2 – Initial distribution of solutions

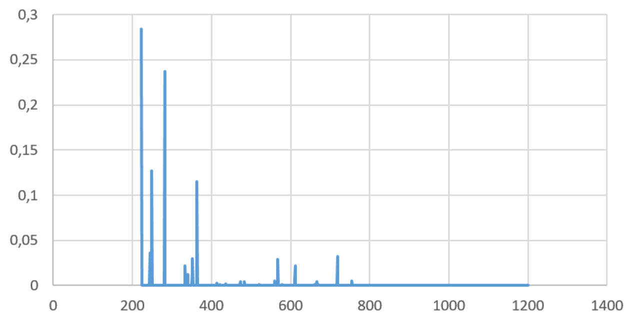

FIG. 3 –Final distribution of solutions

3 Conclusions et perspectives

In summary, we have introduced the IQAOA approach, which leverages an indirect representation of permutations. Our results offer valuable insights into the performance of IQAOA and suggest promising strategies for its practical implementation on near-term quantum devices.

To the best of our knowledge this is the first quantum resolution of a 9 customers TSP with a version of QAOA.

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