Heuristic and exact algorithms for a vehicle routing problem with route cost equity constraints

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1 Introduction

Workload equity is an emerging topic in the routing literature. The challenge is to propose models and algorithms capable of finding solutions with an equitable workload distribution between drivers with a low impact on the routing cost [2]. In the current literature, most papers naturally manage equity within a bi-objective model. In this work, we rather use the classic min cost objective as the single objective and manage equity on the drivers route cost with equity constraints.

We use a test-bed problem selected in the field of healthcare logistics, the Multi-Trip Vehicle Routing Problem with Mixed Pickup and Delivery, and Release and Due dates (MTMPD-RD) introduced in [1].

2 Measure of equity and solution methods

2.1 Equity constraints

The proposed equity constraints consist in limiting the deviation of each route cost from the average route cost. For a constant-sum equity metric (the sum of workload assigned to drivers remains constant for any feasible solution), the average is known. But these constraints are more difficult to manage with a variable-sum metric (the sum of workload assigned to drivers differs between solutions) as the average is unknown and depends directly on the solution. Route cost corresponds to the driving time (service times are not counted) *i.e.*, $c_{ij} = t_{ij}$ and is a variable-sum equity metric.

Let $\mathcal{M}$ denotes the set of drivers and K the number of drivers. Let c^k be the non-negative variable equals to the cost of route of driver m_k. We denote C, the routing cost of the solution ($C = \sum_{m_k \in \mathcal{M}} c^k$). Hence, equity constraints impose for each driver route, a limit deviation above the average routing cost as follows :

$$c^k \leq \frac{\alpha}{K} \times C, \ \forall m_k \in \mathcal{M} \ (\alpha > 1) \tag{1}$$

2.2 Algorithms

We show that integrating constraints (1) within a straightforward set partitioning formulation can not be solved efficiently by a standard branch-and-price approach and propose three new solution methods : a heuristic and two branch-and-price algorithms based on new models to manage the equity constraints.

2.2.1 Heuristic

The heuristic is based on a dichotomic search where at each step, a simplified version of the problem is solved to optimality with a branch-and-price algorithm ; the structure of the solution guides the search for the next step. Although heuristic, in some specific cases, the algorithm proves optimality.

2.2.2 Driver-indexed branch-and-price

The originality of this branch-and-price algorithm is that columns in the column generation are indexed by route and driver. We distinguish the sets of routes between drivers and denote $\Omega_{(mk)}$ the set of feasible routes of driver m_k. So, given a driver m_k and a route $r \in \Omega_{(mk)}$, the decision variable θ_r^k equals 1 if the route is selected for this driver, 0 otherwise. The equity constraints are managed in the master problem :

$$\sum_{r \in \Omega_{(m_k)}} c_r \theta_r^k \leq \frac{\alpha}{K} \times \sum_{m_l \in \mathcal{M}} \sum_{r \in \Omega_{(m_l)}} c_r \theta_r^l, \ \forall m_k \in \mathcal{M} \tag{2}$$

This requires adaptations at each level of the algorithm : the master problem, the column generation, the pricing problem and the branching rules. A weakness of this model is that it requires to solve one pricing problem per driver.

2.2.3 Node-based branch-and-price

Equity concerns drivers so, intuitively equity constraints are expressed on drivers in the previous models. However, they can indifferently be written on nodes instead. The principle is that the cost of the routes served by each driver is limited to $\alpha\times$ the average route cost. This constraint can similarly be stated on the nodes (customers) : given a node, the cost of the route serving this node is limited to $\alpha\times$ the average route cost :

$$\sum_{r \in \Omega} a_i^r \theta_r c_r \leq \frac{\alpha}{K} \times \sum_{r \in \Omega} \theta_r c_r, \ \forall i \in \mathcal{N} \tag{3}$$

Expressing equity constraints that way allows solving a single pricing problem at each step of the column generation instead of one per driver.

3 Computational experiments

Experiments are conducted on instances of [1] which defines a benchmark of realistic instances extracted from the city of Aix-en-Provence, France. Instances are divided into two sets : S_{25} and S_{50}, both containing 30 instances of 25 and 50 customers respectively. To evaluate the impact of equity on cost, different α values are tested : $\alpha \in \{1.1, 1.08, 1.06, 1.04, 1.02, 1.01\}$.

The driver-indexed branch-and-price and the node-based branch-and-price showed their limits on solving the 25-customers instances. So, the 50-customers instances are solved only with the heuristic. Results show that managing equity this way permits to find equitable solutions with a small impact on the total routing cost.

Références

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