A Quantum Annealing Solution to the Job Shop Scheduling Problem with Availability Constraints

Riad Aggoune[1], Samuel Deleplanque[2]

[1] ITIS department, Luxembourg Institute of Science and Technology, Luxembourg
riad.aggoune@list.lu

[2] CNRS, Centrale Lille, JUNIA, Univ. Lille, Univ. Valenciennes, UMR 8520 IEMN, 41 boulevard Vauban, Lille Cedex 59046, France.
samuel.deleplanque@junia.com

1 Introduction

Quantum optimization is an emerging and fast developing research field. It consists in using quantum computers and algorithms to tackle NP-Hard problems. This generally requires formulating the problems as Quadratic Unconstrained Binary Optimization (QUBO) models. While current quantum computers are limited in the number of qubits and thus in the number of variables they can handle, it is challenging to build efficient models for optimization problems that integrate practical constraints. The first quantum annealing solution for the job shop scheduling problem with the makespan objective was developed in [4]. The proposed model was extended to the flexible job shop scheduling problem in [3].

We consider in this paper the job shop scheduling problem with availability constraints. We propose a quantum annealing solution for both cases where unavailability periods are fixed and flexible. We first present a mathematical formulation of the job shop scheduling problem as a QUBO model [1]. Then, we show how to integrate the availability constraints to this model. Results of numerical experiments made on the D-Wave quantum annealing computer show the efficiency of the approach.

2 Problem definition

The job shop scheduling problem with availability constraints can be stated as follows : A set of n jobs $J = \{J_1, J_2, \ldots, J_n\}$ has to be processed on a set of m machines $M = \{M_1, M_2, \ldots, M_m\}$. Each job J_i consists in a linear sequence of n_i operations $(O_{i1}, O_{i2}, \ldots, O_{in_i})$. Each machine can process only one operation at a time and each operation O_{ij} with a processing time of p_{ij} time units needs only one machine. There are k unavailability periods $\{h_{j1}, h_{j2}, \ldots, h_{jk}\}$ on each machine M_j. Two cases are considered in the paper : either the starting date S_{jk} of unavailability period h_{jk} of duration p'_{jk} is known in advance and fixed, or it is flexible and can vary within a time window. The objective is to determine the starting date t_{ij} of each operation O_{ij} so that the makespan noted C_{max} is minimized. The job shop scheduling problem with availability constraints is strongly NP-hard since the 2-machine flow shop scheduling problem is strongly NP-hard [2].

3 QUBO formulation

The following QUBO models the total completion time in a job shop with the Objective function (1) to minimize with 3 multipliers, λ_1, λ_2, and λ_3, balancing the relaxation of the processing, resource and precedence constraints, respectively.

$$
\begin{aligned}
&\sum_j \sum_t t.x^t_{n_i j} \\
&+\lambda_1 \sum_j \sum_i (\sum_t x^t_{ij} - 1)^2 \\
&+\lambda_2 \sum_{(i,j,t)\cup(i',j',t')\in T1} x^t_{ij} x^{t'}_{i'j'} \\
&+\lambda_3 \sum_{(i,j,t,t')\in T2} x^t_{ij} x^{t'}_{i+1j} \qquad (1)\\
&\text{with :} \\
&T1 = (i,j,t)\cup(i',j',t') : i,i' = 1..n_i, j,j' = 1..n, (i,j) \neq (i',j'), \\
&M_{ij} = M_{i'j'}, (t,t') \in T^2, 0 \leq t'-t < p_{ij} \\
&T2 = (i,j,t,t') : i = 1..(n_i - 1), j = 1..n, (t,t') \in T^2, t + p_{ij} > t'
\end{aligned}
$$

The boolean variable x^t_{ij} takes the value 1 if the operation i of the job j starts in period t, with $i = 1..n_i$, $j = 1..n$, $t = 1..T$, and takes the value 0 otherwise. $M_{ij}, i = 1..n_i, j = 1..n$, is the required machine for the operation i of the job j.

4 Managing fixed and flexible unavailability periods

We apply a pre-processing on the two types of availability constraints. For cases where the non-availability windows of a machine are known in advance, meaning we already know the periods during which no production operation can be performed, we treat each of those non-availability period as a single operation that has already been scheduled. Then, for each constraint of this type, we create a new job consisting of one operation for which the starting date is fixed. In cases where the non-availability periods of a machine are unknown, we want to minimize an 'extended' makespan. Indeed, if we only consider the makespan as the objective function, the non-fixed period of non-availability might be scheduled after all the production operations are finished (or at at time it does not penalise the operations). However, the problem becomes more meaningful if we consider that all operations and all non-availability constraints must be completed as soon as possible. To manage such new constraints, we create a new job consisting of one operation for which the starting date is S_{jk}. Since we minimize the sum of the ending time of each operations, we can easily integrate the new operations resulting from the non-fixed non-availability periods. We can thus address both types of non-availability in the same problem by creating new jobs consisting of a single operation and by fixing the starting date if the related period of non-availability is known.

Références

[1] Aggoune, R. and S. Deleplanque. A Quantum Annealing Solution to the Job Shop Scheduling Problem. *ICCSA 2023. Lecture Notes in Computer Science*, vol 14104. Springer. https ://doi.org/10.1007/978-3-031-37105-928

[2] Blazewicz and J. Breit and P. Formanowicz and W. Kubiak and G. Schmidt. Heuristic algorithms for the two-machine flowshop Problem with limited machine availability. *Omega Journal*, 29 :599–608, 2001.

[3] Denkena Berend and Schinkel Fritz and Pirnay Jonathan and Sören Wilmsmeier. *Quantum algorithms for process parallel flexible job shop scheduling. CIRP Journal of Manufacturing Science and Technology*, 12142, 2020.

[4] Venturelli, D. and Marchand, D. J. and Rojo, G. *Quantum annealing implementation of job-shop scheduling. arXiv preprint :1506.08479*, 2015.