A Stochastic Model for Controlling Friends and Family Investment in Crowdfunding

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1 Introduction

Crowdfunding has changed how entrepreneurs gather small sums of money from various investors for new projects [1]. This paper specifically examines the crucial role of friends and family (F&F) investments in crowdfunding campaigns. We introduce a mathematical model designed for several functions of current and past investments. What sets our approach apart is its real-world impact, consistently providing results tested using MATLAB simulations. By strategically managing investments time and contributions, creators can control the success of their crowdfunding campaign.

2 Mathematical Model & Simulation Results

2.1 Strategic F&F Investment Model

Our model is designed to strategically manage Friends and Family (F&F) investments in crowdfunding campaigns, aiming for the project success. It operates within a campaign period from time 0 to T, targeting a funding threshold M. At each discrete time step $t \in \{2, \dots, T-1\}$, the random investment $X(t)$ received by the project at time t is given by:

$$X(t+1) = X(t) + Z(t),$$

Or

$$Z(t) \sim P(\mu(t));$$

where $Z(t)$ is a Poisson random variable with parameter

$$\mu(t) := u(t) + f(X(t), X(t-1)),$$

$X(t)$ is used in the expression to denote the cumulative investment up to time t.
Let $u(1), u(2), \dots, u(N)$ be the control inputs representing investments from friends and family at each time. Note that initial investments $X(0)$ is given and usually equals to 0. This model aims to leverage the difference between current and previous investments to strategically impact new stochastic investments.

This choice of modeling $Z(t)$ as a Poisson random variable is grounded in the stochastic nature of crowdfunding investments, characterized by discrete and unpredictable events. The parameter $\mu(t)$ integrates intentional investments from friends and family ($u(t)$) and the influence of past and current cumulative investments ($f(X(t), X(t-1))$). This approach enables the model to adapt to the dynamic nature of crowdfunding campaigns, where the project's success influences the likelihood of future investments.

2.2 Simulation Results

The simulation examines the funding progress of a project over time with characteristics defined by different functions $f(x, y)$.

$$\begin{aligned} \text{For (a): Linear} \quad & f(x,y) = \alpha x + \beta(x - y), \\ \text{For (b): Square-root} \quad & f(x,y) = \alpha x + \beta\sqrt{x - y}, \\ \text{For (c): Indicator} \quad & f(x,y) = \alpha x + \beta \cdot 1_{[\epsilon,\infty)}(x - y), \end{aligned}$$

Where:

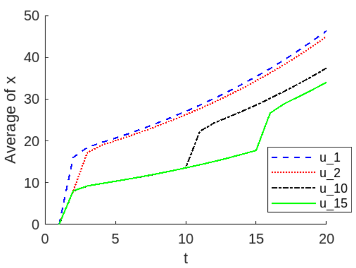
- x represents the funding level of the project at time t,
- y represents the funding level of the project at the previous time step,
- α and β positive parameters.

The exploration assesses the impact of different control strategies (u) on the funding progress. The control strategies considered in the code are as follows:

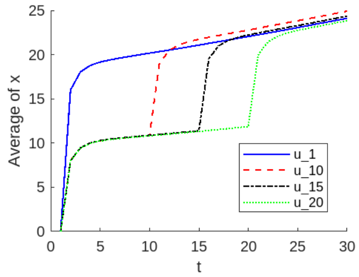
$$u_n(t) = \begin{cases} B & \text{if } t = 1 \text{ and } n = 1 \\ B/2 & \text{if } t = 1 \text{ or } t = n > 1 \\ 0 & \text{otherwise} \end{cases}$$

Where $u_n(t)$ represents the control input for the investment strategy at time t and from source n and B represents the number of friends and family contributors.

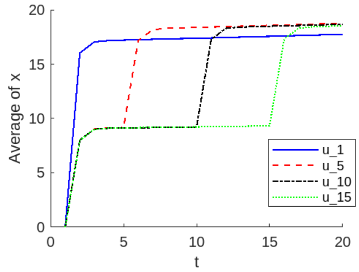
For $n = 1$, the strategy allocates all contributions (B) to the first time slot.
For $n > 1$, the strategy allocates half of the contributions ($B/2$) to the first time slot and the other half to the n-th time slot.
Based on 10K simulations, the average investments dynamics are in the following figures.



(a) Linear function and $\alpha = 5$ and $\beta = 0.1$.



(b) Square-root function with $\alpha = 0.5$ and $\beta = 0.5$.



(c) Indicator function with $\alpha = 0.2$ and $\beta = 1$ and $\epsilon = 2$.

Results illustrate that concentrating contributions at the project's outset ($u_1(t)$),which includes investments at the beginning, achieves better growth compared to other investments and tends to yield the most successful projects for (a). For cases (b) and (c), the simulations indicate that while the strategy of allocating all the money at the beginning of a project may have an initial advantage, it may not always be the best approach in the long term. This result suggests that allocating funds mid-project can have a greater and lasting impact on project success.

3 Conclusion

In conclusion, our research focuses on expanding our analysis to offline policy, specifically in predicting the total pledged amount. Additionally, our MATLAB simulations consistently show that concentrating contributions at the beginning significantly improves our model's performance in linear scenarios. Looking forward, we aim to explore online policy optimization using Markov Decision Processes to make effective real-time decisions during campaigns, aiming for optimal outcomes in crowdfunding campaigns.

References

[1] Agrawal, A. K. and Catalini, C. and Goldfarb, A. *The Geography of Crowdfunding*, National bureau of economic research, Tech. Rep., 2011

[2] Linda Salahaldin, Vineeth S Varma, Salah Eddine Elayoubi. *When and how to intervene for saving an entrepreneur's crowdfunding campaign.* Finance Research Letters, 2022.