Combinatorial Optimization and Machine Learning to build delay resilient aircraft rotations

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Introduction Optimizing aircraft routing is a critical task that airlines must address daily. This process begins one year before operations, when only high-level data such as the flight schedule is available, and continues until the day before operations, when more comprehensive details such as maintenance schedules and crew pairings are known. The goal is to assign an aircraft to each flight leg in a way that minimizes the total operational cost, without violating any operational constraints.

Air France routinely solves this problem using either a Mixed Integer Linear Program or a column generation formulation [3], depending on the size of the subfleet. However, these methods do not sufficiently account for the resilience of the routing solution to delays and the associated costs due to delay propagation along the planned route. Indeed, when delays are taken into account, the objective function becomes non-linear, making the problem significantly more complex to solve.

Given a *leg schedule* x, let $\mathcal{Y}(x)$ be the feasible set of aircraft routing y, such that $y_a^i \in \{0,1\}$ is 1 if connection a is made by aircraft i. The delays $\xi(y)$ occurring along the schedule are an endogenous random variable, depending on the routing decision y. The *stochastic aircraft routing problem* can be formulated as

$$\min_{y\in\mathcal{Y}(x)} \sum_a d_a^i y_a^i + \mathbb{E}[c(\xi(y), y)], \tag{1}$$

with c a cost function of the delay, and d_a^i the operational cost of operating connection a with aircraft i.

Overview of the approach We first propose a method based on a Bayesian Network with continuous variables parameterized by a Neural Network [2] to predict delay distributions for any given leg schedule. These predicted distributions are then used to evaluate solutions of the stochastic aircraft routing problem through Monte Carlo scenario sampling. Finally, we build a heuristic to find resilient and low operational cost routing, using a Combinatorial Optimization enhanced Machine Learning pipeline [1].

Delay model We model the delay propagation along an aircraft routing as a Bayesian Network (see Figure 1), i.e. an acyclic conditional dependence digraph between *observed delay* random variables ξ_ℓ and *intrinsic delay* random variables ε_ℓ. This allows for learning a deep learning predictor φ_w predicting log-normal intrinsic delay distribution parameters for each leg of the schedule.

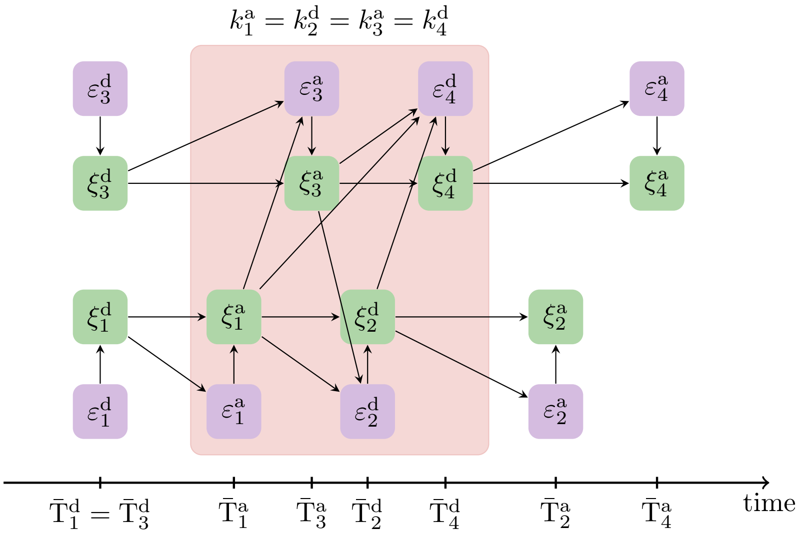

FIG. 1 – Bayesian Network for two aircraft routes with one common airport k in red.

A policy parameterized by a neural network We then model a policy π_w for the stochastic aircraft routing problem as a hybrid pipeline (2) with two layers :

$$\xrightarrow[x]{\text{Instance}} \boxed{\begin{array}{c}\text{Neural}\\ \text{Network } \varphi_w\end{array}} \xrightarrow[\theta\,=\,\varphi_w(x)]{\text{Objective}} \boxed{\boxed{\begin{array}{c}\text{Deterministic}\\ \text{Aircraft Routing } f\end{array}}} \xrightarrow[y\,=\,f(\theta)]{\text{Routing}} \tag{2}$$

- First, a neural network φ_w, the *Machine Learning encoder*, which maps an instance of the stochastic aircraft routing problem to an instance of its deterministic version, by predicting the cost weights θ in the objective function. This layer is parameterized by learnable weights w.
- Then, the *Combinatorial Optimization layer*, that solves the deterministic aircraft routing problem parameterized by θ, and outputs a feasible solution routing y.

Learning the pipeline We learn parameters w of the policy by comparing several state-of-the-art methods :
- Learning by experience by directly minimizing the total cost function (1).
- Learn by imitating anticipative solutions computed using a descent heuristic for a given scenario.

Numerical experiments Extensive numerical results on Air France historical data will be presented for both the delay model and stochastic aircraft routing learning.

Références

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