Online Participatory Budgeting

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1 Introduction

Deciding whether to fund a proposed project from a limited budget or not is one of the most common and crucial problems in basically any organisation. The rise of participatory budgeting (PB) over the last 30 years [Dias et al.(2019)] has demonstrated that such funding decision can be made in a democratic and participatory way. So far, PB has mostly been used on the level of municipalities, but recently it has also been applied in smaller organisations, for example in Lithuanian schools[1]. Currently, PB is usually organized as a yearly one shot event in which the whole budget for the year is decided upon in one election. However, in many cases, funding decisions have to be made continuously throughout the year whenever a new project is proposed. For example, many cities have a fixed yearly budget for improving their infrastructure and during the year have to continuously evaluate different improvement projects to decide whether they should be funded or not. In this paper, we propose a novel framework for making such online funding decisions in a democratic way.

PB has recently received significant attention from the area of (computational) social choice [Rey and Maly(2023)]. However, the existing literature has mainly focused on one-shot PB instances. The only exception is a paper by [Lackner et al.(2021)], which studies sequences of PB elections. However, this is quite different from our model, as [Lackner et al.(2021)] assume that every instance has its own budget, while we assume that the projects are revealed over time but the budget remains the same. Models integrating temporal aspects into voting are more widely studied in approval based committee elections [Lackner and Skowron(2023)], which can be viewed as a special case of PB where all the projects have a unit cost. This includes [Lackner(2020)], [Lackner and Maly(2023)] and [Chandak et al.(2023)] who looked at sequences of approval elections. In contrast to our model, they assume that in each step one of several candidates must be elected, while in our model we have to make a decision for or a against a single candidate in each step. Finally, the models that are most similar to ours are the online committee election model of [Do et al.(2022)] in which candidates are presented in an online setting and we need to decide on selecting them directly after their presentation and the proportionally fair online allocation of public good model of [Banerjee et al.(2022)] where the difference from [Do et al.(2022)] is that the public goods can be partially funded. This is inspired by the multiple secretary problem [Bateni et al.(2013)] and can be seen as the unit-cost special case of our model. Beyond the social choice literature, we want to mention the online knapsack problem introduced by [Marchetti-Spaccamela and Vercellis(1995)], which is closely related to online PB. However, to the best of our knowledge proportionality and fairness, which are the main focus of our study, have not been considered in the online knapsack literature.

We introduce the notations and main definitions in section 2, present our main results in section 3 and conclude in section 4.

[1] `https://transparency.lt/en/topic/participatory-budgeting/`

2 Preliminaries

An instance of Participatory Budgeting (in short PB) is a tuple $I = (\mathcal{P}, c, b)$ where $\mathcal{P} = \{p_1, \dots, p_m\}$ is the set of all projects, $c : \mathcal{P} \to \mathbb{R}_{>0}$ is the cost function and $b \in \mathbb{R}_{>0}$ the budget constraint. Given a subset $P \subseteq \mathcal{P}$, we denote by $c(P) = \sum_{p \in P} c(p)$ the cost of all projects in P. The goal in PB is to find an allocation $\pi \subseteq \mathcal{P}$ that is feasible, i.e., that satisfies $c(\pi) \leq b$. The set of all feasible allocations given an instance I is denoted $F_{EAS}(I) = \{\pi \subseteq \mathcal{P} \mid c(\pi) \leq b\}$.

Let $\mathcal{N} = \{1, \dots, n\}$ be the set of voters who voted in our participatory budgeting instance. We consider the case where voters submit approval ballots over the set of projects $\mathcal{P}$. We denote by $\mathcal{A} = (A_1, \dots, A_n)$ the approval ballots, where for each $i \in \mathcal{N}$, $A_i \subseteq \mathcal{P}$ is the set of all projects voter i approves. Given a project $p \in \mathcal{P}$, we denote by $\mathcal{N}(p) = \{i \in \mathcal{N} \mid p \in A_i\}$ the set of all voters approving project p.

A PB rule R takes as an input a tuple $I, \mathcal{A}$ and returns a set of feasible allocations $R(I, \mathcal{A}) \subseteq F_{EAS}(I)$. These allocations are the co-winners under R.

Let us now introduce our model of online PB. Let $T = \{1, \dots, t\}$ be a set of $t \leq m$ rounds where in each round $i \in T$, a project $p(i) \in \mathcal{P} \setminus \{p(j), j < i\}$ is revealed. An online PB rule is a rule that iterates through T and decides at each round $i \in T$ whether to select $p(i)$ or not without any information on the projects in future rounds. Note that we might have $t < m$.

Definition 1 *An allocation π is priceable if there exist a family of contribution functions $(\gamma_i)_{i \in \mathcal{N}}$ and a "personal budget" α given to all voters s.t.*

- *(C1) Agents contribute only to projects they approve on: $\forall i \in \mathcal{N}, p \notin A_i \Rightarrow \gamma_i(p) = 0$.*
- *(C2) Agents contribute only to projects in π: $\forall i \in \mathcal{N}, p \notin \pi \Rightarrow \gamma_i(p) = 0$.*
- *(C3) No agent contributes more than α: $\forall i \in \mathcal{N}, \sum\limits_{p \in \pi} \gamma_i(p) \leq \alpha$.*
- *(C4) Projects in π receive enough contribution to be funded: $\forall p \in \pi, \sum\limits_{i \in \mathcal{N}} \gamma_i(p) \geq c(p)$*
- *(C5) No group of agents supporting a non-selected project $p \in \mathcal{P} \setminus \pi$ is left with more than $c(p)$: $\forall S \subseteq \mathcal{N}, p \in \bigcap\limits_{i \in S} A_i, \sum\limits_{i \in S} (\alpha - \sum\limits_{q \in \pi} \gamma_i(q)) < c(p)$*

We say that a family of contribution functions (or price system) is reasonable when it satisfies (C1) to (C4).

3 Axiomatic Approach

[Do et al.(2022)] proposed an algorithm called the Greedy Budgeting rule for online committee elections, we can use this algorithm also for online participatory budgeting. We divide the budget b among the voters, each of them initially has b/n dollars. When a project p appears, we check if the voters who approve p have at least $c(p)$ dollars in total. If so, we divide its cost among its supporters (the partition of costs is not specified in this algorithm) and select this project. Let us consider an example illustrating the Greedy Budgeting rule.

Example 1 *Let us consider the online participatory budgeting instances depicted in table 1 with a budget $= 6000$. We assume projects are presented in increasing order of their indices, i.e., first p_1, then p_2 and so on. Note that each voter initially gets a budget of $\frac{6000}{3} = 2000$. If we apply the Greedy Budgeting rule and use a contribution function that is as fair as possible as suggested by [Do et al.(2022)], we end up selecting only project p_2, having voters 1 and 3 pay 1000 each for this project. It is straightforward to check that this allocation is priceable.*

In the example above, the Greedy Budgeting rule produced a priceable outcome. As it turns out, this is always the case.

project	1	2	3	price
p_1	1	0	1	5000
p_2	1	0	1	2000
p_3	0	1	1	4000
p_4	1	0	0	1000

TAB. 1: The participatory budgeting instance used in Example 1

project	1	2	3	price
p_1	0	0	0	5000
p_2	2000	0	0	2000
p_3	0	2000	2000	4000
p_4	1	0	0	1000

TAB. 2: The price system supporting $\{p_2, p_3\}$

Theorem 1 *The Greedy Budgeting rule satisfies priceability.*

Proof : Let $(\mathcal{P}, c, b), \mathcal{A}$ be a participatory budgeting instance, and $\pi \subseteq \mathcal{P}$ be an allocation given by the Greedy Budgeting rule from some order of presentation of projects. This rule already gives us a collection $(\gamma_i)_{i \in N}$ of contribution functions. One can easily check that (C1) to (C4) are met by $(\gamma_i)_{i \in N}$. Suppose there is a group of agents $S \subseteq N$ supporting some project $p \in \mathcal{P}$ which is left with more than $c(p)$ at the end of the algorithm. Then, as the voter's budgets are non-increasing trough the algorithm, the voters in S must have had at least $c(p)$ together in the round $i \in T$ in which p was presented. Therefore, the Greedy Budgeting rule would have selected this project p when it was presented. $\square$

However, priceability is a very weak requirement. Indeed, the allocation produced by the Greedy Budgeting rule in Example 1 is priceable, yet it is not efficient since it is also possible to fund project p_3 (see table 2) or project p_4 that appeared after p_2 and have an allocation that is priceable. In order to do so, we just need to reallocate the cost of projects among their supporters like in the maxmin rule introduced by [Sánchez-Fernández et al.(2022)]. In order to find an efficient re-allocation of project costs, we use a max flow approach.

Given a subset $P \subseteq \mathcal{P}$, we define a flow network $G_P = (V_P, E_P, k_p)$ where E_p is the set of edges, V_p the set of vertices and $k_P : E_P \to \mathbb{R}^+$ the capacity function as follows. $V_p = \{s, t\} \cup P \cup N$, for $p \in P, i \in N, (p, i) \in E_P$ iff $p \in A(i)$ and $k_P(p, i) = b/n$, also $(s, p) \in E_P$ with $k_P(s, p) = c(p)$ and $(i, t) \in E_P$ with $k_P(i, t) = b/n$. Vertices s and t are respectively the source and the sink of this flow network.

Using such flow networks, we can define an adaptation of the Greedy Budgeting rule called Efficient Greedy Budgeting rule: The Efficient Greedy Budgetuing rule adapts the contribution function at each step in order to have a more efficient outcome. When a project p is shown to the voters, given a previous allocation $\pi \subseteq \mathcal{P}$, if $c(p) + c(\pi) \leq b$ we solve the Max Flow problem on $G_{\pi \cup \{p\}}$ using the capacity scaling algorithm from [Ahuja and Orlin(1995)]. If the maximum flow of this network has a value $c(\pi) + c(p)$ (it cannot exceed this value due to the capacity of edges), we select p and use the contribution function induced by this flow, otherwise we keep the previous allocation and contribution function. This rule resembles the maximin rule introduced by [Sánchez-Fernández et al.(2022)] as it allows for re-allocations of the costs of selected projects between the supporting voters.

In Example 1, the efficient greedy budgeting rule selects both p_2 and p_3 with the price system described in table 2 while the greedy budgeting rule selected only p_2 and was then less efficient. We introduce a new strengthening of priceability which aim is to cover the cases that are similar to our example, while being statisfiable.

Definition 2 *An allocation π is E-priceable if it is priceable, and every reasonable family of contribution functions that supports π satisfies (C5).*

Theorem 2 *The Efficient Greedy Budgeting rule satisfies E-priceability and has a complexity of $O(np^2 \log(b))$.*

Proof : The capacity scaling algorithm from [Ahuja and Orlin(1995)] runs in $O(E\log(C))$ where E is the number of edges and C the maximal capacity. In the flow network we use, we have $E \leq |\pi| + n + n \times |\pi| = O(np)$ and $C \leq b$. As we use the capacity scaling algorithm at most p times, our global complexity is in $O(np^2 \log(b))$.

Let π be an allocation given by our algorithm, and F be a maximal flow of G_π, we will study this flow on $G_\mathcal{P}$. We define a family of contribution functions $(\gamma_i)_{i\in N}$ as $\forall i \in N, p \in \mathcal{P}, \gamma_i(p) = F(p,i)$ (by abuse of notation, we set $F(p,i) = 0$ for $(p,i) \notin E_\mathcal{P}$). We claim that this contribution function satisfies (C1)-(C4).

As $\forall i \in N, p \in \mathcal{P}, (p,i) \in E_P$ *iff* $p \in A(i)$, voters only contribute to projects they approve. Hence, (C1) is satisfied. Also, as F is a flow defined in G_π, voters only contribute to projects in π, as required by (C2). Moreover, as F is a feasible flow and $\forall i \in N, k_\pi(i,t) = {}^b\!/\!_n$, no voter contributes in more than ${}^b\!/\!_n$, satisfying (C3).

As π was selected by our algorithm, we must have that $\sum_{p\in\pi} F(s,p) \geq c(\pi)$ and the feasibility of F imposes $\sum_{p\in\pi} F(s,p) \leq \sum_{p\in\pi} k_P(s,p) = c(\pi)$. As $\forall p \in \mathcal{P}, F(s,p) \leq c(p)$ and $\sum_{p\in\pi} F(s,p) = \sum_{p\in\pi} c(p)$, we obtain $\forall p \in \pi, \sum_{i\in N} \gamma_i(p) = F(s,p) = c(p)$, so all projects in π receive enough contribution to be funded. Therefore, (C4) is also satisfied.

From now on, let γ be any reasonable family of contribution functions. Suppose there is a group of voters $S \subseteq N$ and a project $q \in \mathcal{P} \setminus \pi$ s.t. $\forall i \in S, q \in A(i)$ and $\sum_{i\in S}({}^b\!/\!_n - \sum_{p\in\pi} \gamma_i) \geq c(q)$. In other words, the group S is left with more than $c(q)$. Then, there exists a family of contribution functions $(\gamma'_i)_{i\in S}$ s.t.

$$\forall i \in S, \gamma_i(\pi) + \gamma'_i(q) \leq {}^b\!/\!_n \text{ and } \sum_{i\in S} \gamma'_i(q) = c(q).$$

Let $\pi' \subseteq \pi$ be the current allocation when project q was presented. Consider the network $G_{\pi'\cup\{q\}}$. We define the flow F' as

$$\forall p \in \pi', F'(s,p)_{G_{\pi'\cup\{p\}}} = F(s,p)_{G_\mathcal{P}}, \forall i \in N, F'(p,i)_{G_{\pi'\cup\{p\}}} = F(p,i)_{G_\mathcal{P}}, F'(i,t)_{G_{\pi'\cup\{p\}}} = F(i,t)_{G_\mathcal{P}}.$$

Furthermore, we define $F^{(2)}$ as $F^{(2)}(s,q) = c(q), \forall i \in S, F^{(2)}(q,i) = \gamma'_i(q) = F^{(2)}(i,t)$. One can check that $F^{(3)} = F' + F^{(2)}$ is a feasible flow, and that $\sum_{p\in\pi'\cup\{q\}} F^{(3)}(s,p) = c(\pi') + c(q)$. Hence, our algorithm would have selected q at this step as the max flow had a value of $c(\pi') + c(q)$. By contradiction, we have proved (C5). It follows that π is E-priceable as it is supported by at least one price system and every reasonable price system supporting it satisfies (C5). □

Can we also satisfy other axioms? One natural candidate could be the axiom *local fair share*, which was recently introduced by [Maly et al.(2023)]. It is based on the notion of share:

Definition 3 *We say the* share *of a voter i from allocation π is*

$$share_i(\pi) = \sum_{p\in\pi\cap A_i} \frac{c(p)}{|\mathcal{N}(p)|}.$$

In other words, the share of a voter from a single project is just the cost of the project, equally divided among its supporters. Moreover, we say the fair share *(FS) of a voter i is defined by $FS(i) = \min({}^b\!/\!_n, share_i(A_i))$. Intuitively, the fair share of a voter is ${}^b\!/\!_n$ unless they do not approve sufficient projects to receive a share of ${}^b\!/\!_n$ even if all projects they approve are funded.*

Ideally, every voter should receive their fair share. However, it is easy to see that this is not possible in general. Therefore, [Maly et al.(2023)] introduced the concept of local fair share.

project	1	2	price
p_1	1	0	$b/2$
p_2	1	1	$b/2$
p_3	0	1	ϵ
p_4	0	0	b

TAB. 3: The instance used in the proof of Proposition 1

Definition 4 *We say an allocation π satisfies* local fair share *(local-FS) if there is no project $p \in \mathcal{P} \setminus \pi$ such that for all $i \in \mathcal{N}(p)$ we have*

$$share_i(\pi \cup \{p\}) < FS(i).$$

Consider the Greedy Share rule, which is defined as follows: We initiate $\pi = \emptyset$, when presented with a project p, we check whether $share_i(\pi \cup \{p\}) < FS(i)$ holds for all $i \in \mathcal{N}(p)$. If yes, then we accept p otherwise we reject it. It is straightforward to see that the Greedy Share rule satisfies local FS.

We introduce an impossibility theorem, which highlights the incompatibility of fairness and efficiency in the online PB setting.

Proposition 1 *No online PB rule can always satisfy both priceability and local fair share.*

Proof :

Consider the participatory budgeting instance depicted in table 3 and further assume $0 < \epsilon < b/4$. We build the following adaptive three-round instance: the first project to appear is p_1. If it does not get accepted, we present project p_4 twice and our algorithm would fail priceability. Otherwise, we present p_2. Again if the algorithm does not select it we present p_4 afterwards and it would fail priceability. If both p_3 and p_2 are selected, we present p_1 which is impossible to fund as the budget constraint would be violated. Then, we have $share_2(\{p_1, p_2, p_3\}) = b/4 + \epsilon < FS(2) = b/2$. Thus, the algorithm fails local fair share. □

Corollary 1 *No online PB rule can always satisfy both E-priceability and local fair share.*

4 Conclusion

We have established a novel framework for online participatory budgeting and presented some first results, showing that one cannot expect to satisfy priceability and local fair share together while both are satisfied by simple greedy algorithms. Additionally, we introduced a new strengthening of priceability that guarantees that outcomes are, in a certain sense, efficient. In the future, we want to explore whether we can further strengthen priceability or local fair share and under which conditions it might be possible to satisfy the two axioms together.

Moreover, we plan to work on approximation algorithms for priceability and fair share that can find a good compromise between both properties. In order to do so, one could explore set-aside greedy algorithms that are used in [Do et al.(2022)] and [Banerjee et al.(2022)] or model the problem as a LP like it is done by [Sánchez-Fernández et al.(2022)] in the offline setting and use a primal-dual approach [Buchbinder et al.(2009)].

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