Recurrent Neural Networks for Geometric Optimization with Joint Probabilistic Constraints

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1 Introduction

Zener [1] initiated exploration into minimizing costs in engineering design problems, sparking what is now recognized as geometric programming (GP). Zener's contributions, coupled with subsequent papers by Duffin and Peterson [2], established the essential foundations of this discipline. The term *geometric programming* emerged due to the substantial influence of the arithmetic-geometric mean inequality during its early evolution. Initially, geometric programming primarily minimizes posynomial functions while upholding inequality constraints. A general form of a geometric program can be then stated as follows

$$\min_{x\in\mathbb{R}^M_{++}} \sum_{i\in I_0} c_i \prod_{j=1}^{M} x_j^{\alpha_{ij}}, \quad \text{subject to} \qquad \sum_{i\in I_k} c_i \prod_{j=1}^{M} x_j^{\alpha_{ij}} \leq 1, \;\; k=1,....,K, \tag{1}$$

where x is a strictly positive M-dimensional vector, the exponents α_{ij} are arbitrary real numbers and the coefficients c_i are positive. This work focuses on solving geometric programs with joint probabilistic constraints using recurrent neural networks.

2 A recurrent neural network approach

In this work, we first the following stochastic GP problem

$$\min_{t\in\mathbb{R}^M_{++}} \quad \mathbb{E}\left[\sum_{i\in I_0} c_i \prod_{j=1}^{M} t_j^{a_{ij}}\right], \tag{2}$$

$$s.t \quad \mathbb{P}\left(\sum_{i\in I_k} c_i \prod_{j=1}^{M} t_j^{a_{ij}} \leq 1, k=1,....,K\right) \geq 1-\epsilon, \tag{3}$$

where $\{c_i\}_{i\in I_k}$, $k \in \{1,..,K\}$ are pairwise independent normally distributed random variables i.e $c_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$, $i \in I_k$ where $\mu_i \geq 0$ is the mean value and σ_i is the standard deviation of c_i. The coefficients $a_{ij}, i \in I_k, j = 1, ..., M$ are deterministic, and $1-\epsilon$ is a given probability level with $\epsilon \in (0, 0.5]$.

We first derive a biconvex equivalent for (2)-(3), then we study the optimality conditions using the partial KKT system. Based on the optimality conditions, we propose a recurrent neural network, see Figure (1), that is stable and converges in the sense of Lyapunov to a solution to the initial problem.

Later, we study the case where $\{c_i\}_{i \in I_k}$, $k \in \{1, .., K\}$ are pairwise dependent and normally distributed random variables using Copula theory.

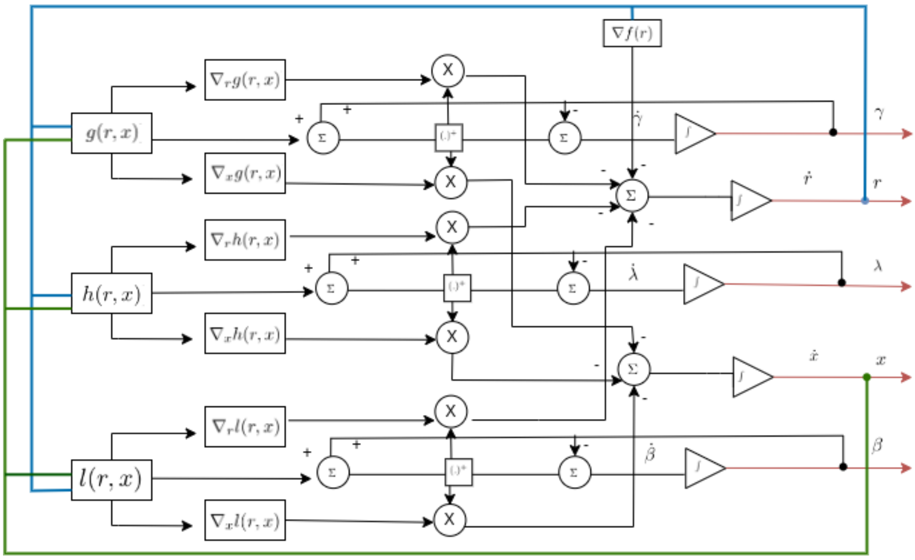

FIG. 1: A simplified circuit implementation of the recurrent neural network

3 Experimental results

To assess the effectiveness of our approach, we examine a transportation problem in three dimensions. Here, the aim is to determine the best form for a transportation box while considering geometric limitations. We evaluate our approach's solution compared to the piecewise linear approximation method presented in [3]. Results show that the recurrent neural network approximates better the optimal solutions and covers well the risk area.

4 Conclusion

This research yields a significant advantage, which is the capability to solve independent and dependent joint chance-constrained geometric programs without relying on convex or linear approximation methods. The numerical experiments showcase that our approach approximates the optimal solution better than the existing state-of-the-art methods. Moreover, it effectively encompasses the risk area by furnishing robust solutions.

References

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