A multi-objective optimization model for opportunistic maintenance of repairable spare parts

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1 Introduction

Closed loop supply chains (CLSC) have become necessary to advance sustainable practices in the medical systems industry [5, 2]. The recovery processes, including repair, remanufacturing, and recycling, have been established as key drivers in reducing the demand for virgin materials, curtailing energy use, and limiting waste, thereby contributing to environmental and economic goals [10].

Traditional maintenance strategies recommend using new spare parts to reduce the likelihood of failure after repair. On the other hand, CLSC considers repaired units to be as good as new ones, which might lead to future failures and limit the circular benefit of this framework. Thus, there is a need to improve repair center operations to balance between reliability and sustainability goals.

Opportunistic maintenance extends maintenance to at-risk components during opportunistic downtime [6]. The literature primarily discusses age-based and block replacement policies [8, 9]. Other papers adopt a single-objective appraoch focusing on minimizing long-run maintenance costs [1]. However, this method often results in excessive usage of components due to the intricate dynamics of CLSC and the severity of failure incidents. Besides, studies are lacking in incorporating inter-component dependencies in OM policies. In addition, evaluating system reliability with interdependent components is challenging [11]. Techniques like fault tree analysis, Markov chain, and copula are commonly used, but they suffer from limitations in complex scenarios [4, 7].

This paper aims to fill this gap by proposing a multi-objective framework that leverages estimating the reliability under dependency of a spare part after repair and its use for maintenance decisions. We use Non-dominated Sorting Genetic Algorithm (NSGA) to suggest proactive replacements during the downtime opportunity of the spare part. The developed model considers maintenance cost, the long-run failure cost, and the environmental cost.

2 Modeling and problem formulation

In the context of a Closed-Loop Supply Chain (CLSC) for medical equipment, we focus on balancing cost and reliability in spare parts management through opportunistic maintenance.

Our model addresses the repair of medical units by considering replacing failed and at-risk components to enhance system reliability. Maintenance decisions are influenced by economic, stochastic, and structural dependencies, affecting costs, failure rates, and maintenance procedures.

The model aims to minimize costs, environmental impact, and risk, adhering to a reliability standard. It leverages a single opportunity window for decision-making, reflecting practical constraints in maintenance operations. To formally define the optimization model, we introduce the following notations :

- $\zeta = [1, 2, 3, ..n]$: set of components in the spare part,
- $Cost_c$: price of component c,
- M_c : the average lifetime of component c,
- $RV_c = \frac{cost_c}{M_c}$: residual value of component c,
- LC : labor cost,
- $Cost_0$: logistic cost for each repair (shipping cost to replace the LRU with a new one at the client site),
- τ_c : disassembling time for component c,
- a_c : age of component c,
- $R_c(t)$: reliability function of component c,
- $f_c(t)$: probability density function of failure time for component c,
- $R_{sys}(t; a_1, a_2, .., a_n) = h(R_1(t; a_1), ..., R_n(t; a_n))$: reliability function of the spare part as a function of reliability of its components,
- $f_{sys}(t; a_1, a_2, .., a_n)$: probability density function of system's lifetime,
- T : planning horizon,
- r : interest rate,
- $D = (D_{ij})_{CXC}$: disassembly matrix for the system,
- s_c : state of component c, $s_c = \begin{cases} 1, & \text{if component } c \text{ is in a failing state,} \\ 0, & \text{otherwise.} \end{cases}$,
- S_{price} : revenue from selling the spare part,
- R_{min} : minimum required reliability.

One of the characteristics of spare parts reparation is that the components may have different ages with a large variance. Therefore, estimating the unit reliability is not straightforward. We propose to express the unit's reliability R_{sys} as a function of components' reliability and ages. For example, for multi-independent units in series, the reliability of the part can be expressed as $R_{sys}(t) = \prod_{c \in \zeta} R_c(t; a_c)$.
Decision variables :we define the binary decision variable x_c for each component c, with $x_c = \begin{cases} 1, & \text{if component } c \text{ is replaced preventively,} \\ 0, & \text{otherwise.} \end{cases}$
Constraints : The model incorporates constraints that ensure failed components are replaced correctively (1) and the positive operational benefit (2).

$$x_c + s_c \leq 1, \quad \forall c \in \zeta. \tag{1}$$

$$S_{price} - C_{maintenance} > 0. \tag{2}$$

Objective Functions : The multi-objective optimization problem aims to minimize the following costs :
Maintenance Cost ($C_{\text{maintenance}}$) : This cost includes expenses due to corrective and preventive replacements as well as labor costs involved in the maintenance process :

$$C_{\text{maintenance}} = C_r + C_L, \tag{3}$$

where $C_r = \sum_{c \in \zeta}(x_c + s_c) \times Cost_c$. is the replacement cost calculated by summing the costs of components that are replaced either correctively or preventively. $C_L = 2 \times LC \times \tau_{group}$ represents the labor cost, which is proportional to the disassembly time required to replace the components. We use an approach developed by [3] to calculate the total maintenance time for a component group using the structure connection between components. The cumulative disassembling time of a component c, denoted by τ_c^D, can be defined as the sum of disassembling times for all the components on the path of disassembly (Eq. (4)).

$$\tau_c^D = \sum_{k \in \zeta} \tau_k \times D_{c,k}. \tag{4}$$

For a group of components, there may be some intersections between the disassembly path of different items. As a result, the disassembly duration of the intersection nodes must be counted only once, even if it appears on several ones. Eq. (5) gives the total disassembly time, denoted by τ_{group}, of the replaced components :

$$\tau_{group} = \sum_{c\in\zeta}(s_c + x_c) \times \tau_c^D - \sum_{c\in\zeta} \tau_c^D \times max(\sum_{k\in\zeta}(s_k + x_k) \times D_{k,c} - 1, 0), \tag{5}$$

the first term represents the total disassembly duration of all replaced components when replaced separately ; the second term is the time saving due to intersections among the disassembly paths. In case there is no intersection, the second part in Eq. (5) equals to zero.
Environmental Impact ($C_{\text{environment}}$) : The environmental cost is associated with the waste of unused remaining life of components that are replaced preventively :

$$C_{\text{environment}} = \sum_{c\in\zeta} x_c \times \frac{RV_c}{R_{sys}(0; a_1(1-s_1), .., a_n(1-s_n))} \times \int_0^{+\infty} t f_c(t; a_c)\, dt. \tag{6}$$

Risk Cost (C_{risk}) : The risk cost accounts for potential failures within the planning horizon. When a failure occurs, the logistic cost $Cost_0$ must be counted. However, to compare the future payment to the present time, its *present value* must be calculated.
The age of the replaced components ($x_c + s_c = 1$) would be restored to zero, while it won't change for the other components. So for a small variation of time, the probability of failure can be expressed using the calculated probability density function (*pdf*) of the system f_{sys} and the present value of the logistic cost is $Cost_0 \times (1 + r)^{-t}$. Thus, the total present value of the expected failure cost during the planning horizon, C_f, can be expressed as

$$C_{risk} = \frac{Cost_0 \times \int_0^T f_{sys}(t; a_1(1-(x_1+s_1)), .., a_n(1-(x_n+s_n)) \times (1+r)^{-t}\, dt}{R_{sys}(0; a_1(1-(x_1+s_1)), .., a_n(1-(x_n+s_n))}. \tag{7}$$

Reliability Deviation ($R_{\textbf{deviation}}$) : This metric quantifies the deviation of the part's reliability from the required minimum. The reliability of the part is defined as the probability of surviving the warranty period $T_{warranty}$ given the components' age after repair.

$$R_{deviation} = 100 \times \frac{\max(R_{min} - \frac{R_{sys}(T_{warranty}; a_1(1-(x_1+s_1)),..)}{R_{sys}(0; a_1(1-(x_1+s_1)),..)}, 0)}{R_{min}}. \tag{8}$$

3 System reliability modeling considering stochastic dependency

Accounting for stochastic dependencies in a multi-component system is crucial. To tackle the computational challenge associated with high-dimensional problems, we employ a dimensionality reduction technique alongside Nataf's transformation to compute joint distribution efficiently.
We begin by forming clusters of components based on calculated correlation from the historical replacement of component data. We use Agglomerative Hierarchical Clustering. Each cluster is treated as a 'super component', with its reliability distribution constructed assuming that any component's failure leads to its entire cluster's failure. We use an index similar to the Akaike Information Criterion to determine the optimal number of components per group, aiming to minimize this number while maximizing system reliability.
After clustering, Nataf transformation maps the joint distribution of component groups to a standard normal space. The joint probability density function for a group of components in a serial system, represented by the survival function $R_g(t)$ can be expressed as follows :

$$\begin{aligned} R_g(t) &= P(min_{c\in g}(X_c) \geq t) \\ &= 1 + \sum_{1\leq k\leq n_g} (-1)^k \sum_{1\leq c_1\leq c_2 ... \leq c_k \leq n_g} P(X_{c_1}, X_{c_2}, ..., X_{c_k} \leq t) \end{aligned} \tag{9}$$

This approach simplifies the computational process and enables system reliability evaluation for high-dimensional problems involving multiple dependent components.

4 Industrial case study

In partnership with GE Healthcare, a leader in medical system services, this study applies the OM optimization model to MRI machine power supplies within a CLSC. The model addresses the criticality and costliness of medical device components by minimizing product failure rate and unavailability. We focus on a power supply composed of 11 serially connected components, with maintenance informed by the physical structure and disassembly times (see Figure 1 and Table 1). Table 2 summarizes each component's material cost and average lifetime.

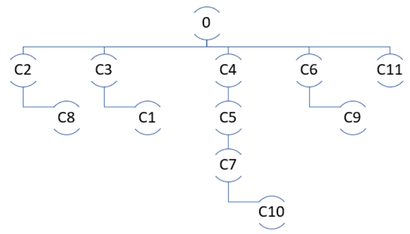

FIG. 1 – System's structure

Component	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**
Disassembling time (U.T)	3	1	1.5	0.2	2	4.5
Component	**C7**	**C8**	**C9**	**C10**	**C11**	
Disassembling time (U.T)	9	4.5	1	1	1	

TAB. 1 – Components' dismantling time

Component	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
Component cost (U.C)	22	24	6	43	140	2	34	23	6	9	8
Mul (U.T)	71k	22k	54k	44k	1.5k	16k	68k	183k	37k	58k	24k

TAB. 2 – Costs parameters

Maintenance data was collected from GEHC's repair center, which manages a large inventory of MRI power supplies, reflecting the challenge of tracking part lifetimes across a global supply chain. Our dataset consists of 13200 records for 7514 parts, with 3490 having multiple repairs. A subset of 120 units with multiple repairs was used to implement the model over a 730 U.T horizon, with the remaining data constructing the reliability functions. We used NSGA algorithm because of the problem's NP-hard and non-linear nature. The optimization seeks to enhance the CLSC's efficiency by balancing the trade-offs between maintenance costs, environmental impact, risk, and reliability.

5 Results and analysis

This section reports the results of our opportunistic strategy on the test subset. First, we present the performance of the clustering method described to model dependency. Then, we present the trade-off between different objectives and their impact on decisions. Finally, we present the opportunistic maintenance effect on parts' reliability.

Reliability functions under dependency : To build system reliability model under dependency, we used the grouping algorithm presented in Section 3. First, we determine the best grouping strategy by maximizing the likelihood and minimizing the number of components per group. The results are given in Table 3, where threshold and groups represent the minimum correlation level selected and the formed groups, respectively ; **N_super_comp** and **Max_comp_group** are the number of formed groups and the maximum number of components per group. It can be seen from Table 3 that the best threshold for grouping components is 0.6 because it maximizes the loglikelihood and minimizes the number of components per group. It can be verified that by grouping like this, the resulting marginal distributions can satisfy the positive definite constraint needed for applying Nataf transformation.
The system reliability is then calculated based on Eq. (9) considering the stochastic dependencies and the forming clusters. Figure 2 compares the result to the empirical estimations directly

Threshold	Groups	N_super_comp	Max_comp_group	Positive definite	AIC	Loglikelihood
0.6	(C1,C3)	10	2	1	10571.19	-10567.19
0.1	(C1,C2,C3,C7,C10)	7	5	0	10578.31	-10568.31
[0.4,0.5]	(C1,C3) ; (C7,C10)	9	2	1	10583.59	-10579.59
[0.2,0.3]	(C1,C2,C3) ;(C7,C10)	8	3	0	10591.78	-10585.78
[0.7,0.9]	(C1) ;(C2) ;(C3) ;(C4) ;(C5) ;(C6) ;(C7) ;(C8) ;(C9) ;(C10) ;(C11)	11	1	1	10593.28	-10593.28
0	(C1,C2,C3,C4,C7,C10) ; (C5,C8,C11) ; (C6,C9)	3	6	0	26482.69	-26470.69

TAB. 3 – Grouping components results

from data. As we can see, the computed system lifetime distribution from the proposed model fits well with the empirical data.

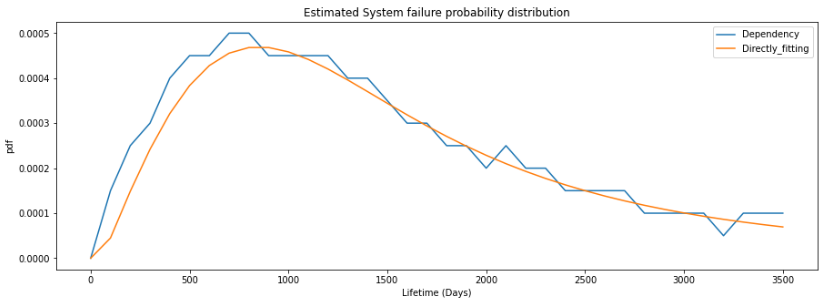

FIG. 2 – Modeling failure distribution under dependency

Pareto Frontier Analysis and Decision-Making Insights : The radar charts in figure 3 portray the Pareto optimal solutions derived from our multi-objective optimization framework for one spare part repair. Each axis on the radar chart quantifies an objective : maintenance costs ($C_{\text{maintenance}}$), environmental impacts ($C_{\text{environment}}$), risk levels (C_{risk}), net benefits, the number of total replacements, and regulatory compliance penalties (R_{penalty}).

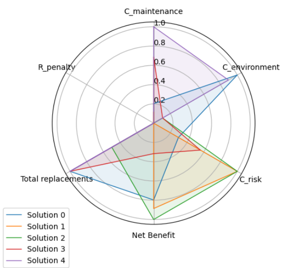

FIG. 3 – Pareto frontier for study case

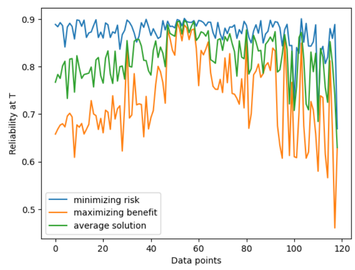

FIG. 4 – Impact of OM strategy on reliability at $T_{warranty}$

It illustrates the balancing act between competing objectives in opportunistic maintenance, with certain solutions favoring financial gain at the potential cost of increased risk, while more balanced profiles suggest a strategic compromise. The model offers valuable managerial insights, advocating for a multifaceted maintenance approach that accommodates profit goals, environmental sustainability, and risk aversion. It underlines the importance of risk management and regulatory compliance, guiding decision-makers toward a comprehensive strategy harmonizing with a wide range of operational goals. It also highlights the complexity of maintenance strategy decisions, providing a tool for identifying an optimized path that integrates diverse priorities.

Impact on the reliability of the repaired spare parts : Figure 4 compare the reliability at the end of the warranty period $T_{warranty}$.Proactive maintenance strategy choices directly impact component reliability as warranty periods end, with the optimal approach depending on management's risk appetite. A benefit-maximizing strategy might meet minimum quality constraints but carries the risk of reduced reliability post-warranty. On the other hand, a risk-minimizing approach promises maximum reliability at the expense of lesser benefits. A balanced strategy strikes a middle ground, offering reasonable reliability and benefits, and is ideal for managers with intermediate risk tolerance. Decision-makers must weigh the long-term effects of their maintenance strategy on system reliability to ensure sustained performance and avoid increased costs from post-warranty failures.

6 Conclusions and perspectives

The study delivers a multi-objective optimization framework for opportunistic maintenance in Closed-Loop Supply Chains, particularly in medical systems. Utilizing NSGA-III, the framework balances various objectives, including maintenance costs and environmental impact, with real-world validation from GE Healthcare data. The Pareto frontiers generated provide a visual guide for decision-makers to tailor maintenance strategies to their operational goals and risk profiles. The research underscores the long-term reliability implications of maintenance decisions made at the end of warranty periods, offering a critical evaluation between short-term benefits and long-term system robustness. This work sets a new standard for maintenance strategy research and opens avenues for further exploration in various industrial contexts.

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