Compendium of mathematical formulations for heterogeneous multistatic sonar networks configuration

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In the context of airborne Anti-Submarine Warfare (ASW), sonars are used to detect, locate and track underwater targets. In this work, we focus on a particular type of sonar: sonobuoys. These are acoustic buoys dropped from an airborne carrier such as a Maritime Patrol Aircraft (MPA), a helicopter or an Unmanned Aerial Vehicle (UAV). More precisely, a sonobuoy is composed of a floating unit housing the UHF/VHF antenna for aerial communications, a submerged unit with the hydrophones, and a cable linking the surface and lower units.

These buoys are then divided into three main categories which are: transmitter-only (Tx), receiver-only (Rx) and transmitter-receiver (TxRx). Since a sonar system involves both a source and a receiver, it is possible, for example, to form one from the pairing of the source of a Tx buoy and the receiver of a TxRx buoy, assuming mutual compatibility. Moreover, when the source and receiver are colocated (i.e. a TxRx buoy), we refer to monostatism, whereas when they are not colocated, we refer to bistatism. Finally, a Multistatic Sonar Network (MSN) is therefore simply a set of sonar systems in monostatic and/or bistatic configuration resulting from the pairing between the various sources and receivers constituting the sonobuoys spread across the area of interest (highly combinatorial). Figure 1 shows a synthetic and simplified view of the operational context addressed here.

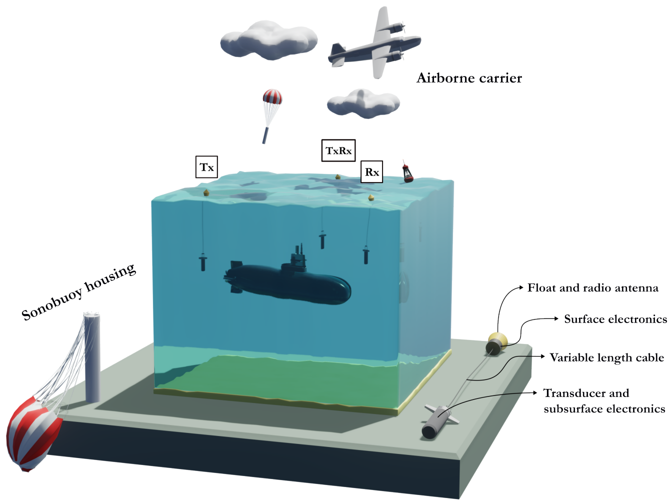

Figure 1: Streamlined illustration of the overall operating environment.

In this work, we will focus on the problem of locating the various buoys forming the MSN in such a way as to maximise the total surface area covered. This is an Area Coverage (AC) problem in the Wireless Sensor Networks (WSN) literature, of which MSNs are a special case [1, 2]. To solve this AC type problem, we will reduce it to a Point Coverage (PC) type problem by discretizing the Area of Interest (AoI) into a set of cells. In the centre of each of these cells, we will then have a deployment position and a fictitious target enabling us to assess the quality of the network. A target is said to be covered (detected), and by extension the cell in which it is located, if the cumulative detection probability exceeds a threshold set beforehand. In this way, the total surface area covered simply corresponds to the percentage of cells covered. Moreover, here, we deal with the case of heterogeneous sensors, which has not yet been addressed in the recent literature [3–7]. In addition, we also optimize the placement of buoys that can be Tx, Rx or TxRx, contrary to the litterature where placement is done on sources (Tx) and receivers (Rx) solely. We will also take into account probabilistic detection models, an adverse effect known as direct blast (masking ellipse) and existing coastlines. These coastlines are retrieved using a Digital Elevation Model (DEM) of the considered AoI, i.e. a discretization of the area in the form of regular cells with bathymetric and topographic data. Due to the intrinsic nature of the buoys, performance between source-receiver pairs varies from one pair to another, and there may be incompatibilities between sources and receivers, for example between high-frequency and low-frequency sonars.

To deal with this problem, we derived 9 mathematical formulations, including 2 base (naïve) formulations and 7 improved formulations resulting from an extension of a linearization of quadratic formulations proposed by Oral and Kettani [8]. Through numerical experiments carried out on 100 instances produced especially for this occasion from real coastal areas, we have identified an ideal model, i.e. one that is significantly better than all the others in the statistical sense.

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