Column generation approach for solving the multi-commodity flow problem with convex objective function

Guillaume Beraud-Sudreau[1], Lucas Létocart[2], Youcef Magnouche[1], Sebastien Martin[1]

[1] Huawei Technologies France, Boulogne-Billancourt, France.
{guillaume.beraudsudreau, sebastien.martin, youcef.magnouche}@huawei.com
[2] LIPN, UMR CNRS 7030, Universite Sorbonne Paris Nord, Villetaneuse, France.
lucas.letocart@lipn.univ-paris13.fr

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1 Introduction

Batch online admission of demands in networks, where re-routing is not allowed, requires routing strategies that leave available bandwidth on each link. The available bandwidth left by the routing of previous batches of demands can be used to route future demand, ensuring an overall efficient use of the network capacity. We will focus on the acceptation of one batch of demands.

Let us consider a graph $G = (V, A)$, where V is the set of nodes and A is the set of links. For each link $a \in A$ we consider a capacity c_a. Let K be a set of demands, where for each demand $k \in K$ we consider a bandwidth d_k, a source s_k, and a sink t_k. A possible approach to solve this problem could be to minimize the Maximum Link Utilization (MLU), as described in the problem below. For each demand, we consider the set of all paths from s_k to t_k denoted P_k.Variable x_k^p represents the percent of traffic from the demand k routed through path p. In Figure 1, constraints (2) represent the convexity constraint ensuring that all demand is served and constraints (3) are the capacity constraints where the MLU is considered.

However, the efficiency of this approach is limited as a single link can drive the whole problem, as illustrated in the example provided in Figure 1. In this toy example, demand 1 has source u_1 and sink u_2 for a bandwidth of 2, demand 2 has source u_3 and sink u_4 for a bandwidth of 1, and demand 3 has source u_3 and sink u_6 for a bandwidth of 1. The couple on the link represents the usage and total capacity of the links. Both solutions are optimal while the one on the bottom leaves more links with free capacity.

$$\min_{\alpha, x_k^p} \alpha \tag{1}$$

$$\sum_{p \in P^k} x_k^p \geq 1 \qquad \forall k \in K, \tag{2}$$

$$\sum_{\substack{k \in K,\ p \in P^k \\ \text{tq. } a \in p}} d_k x_k^p \leq \alpha c_a \qquad \forall a \in A, \tag{3}$$

$$\alpha \leq 1, \tag{4}$$

$$x_k^p \in \{0, 1\} \qquad \forall k \in K, p \in P^k \tag{5}$$

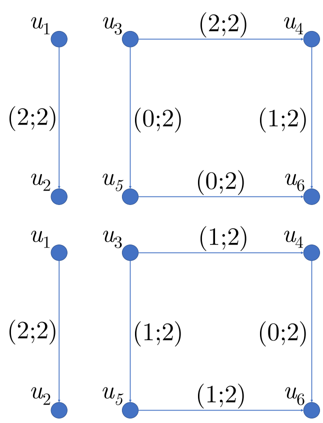

FIG. 1 – A MLU model (left) and two optimal solutions for the same network and set of demand (right).

Better routing strategies can be achieved by solving multi-commodity flow problems minimizing a sum of costs for each link, where each cost is a convex function of the load of the corresponding link. Indeed, as the marginal cost of a link increases with its load, it is less costly to dispatch traffic on different links and leave, as much as possible, capacity available on each link of the network.

In the example above, the second solution would be optimal with convex costs applied to the links while the first one isn't.

2 Problem description

Our goal is to build a multi-commodity flow minimizing convex costs over the links on a network. The extended formulation of the studied problem is :

$$\min_{x_k^p} r(x) = \sum_a r_a\Big(\sum_{\substack{k\in K,\ p\in P^k \\ \text{tq. } a\in p}} d_k x_k^p\Big) \tag{6}$$

$$\sum_{p\in P^k} x_k^p \geq 1 \qquad \forall k \in K, \tag{7}$$

$$\sum_{\substack{k\in K,\ p\in P^k \\ \text{tq. } a\in p}} d_k x_k^p \leq c_a \qquad \forall a \in A, \tag{8}$$

$$x_k^p \in \{0,1\} \qquad \forall k \in K, p \in P^k \tag{9}$$

The objective (6) represents the sum of costs as convex functions of the loads on each link a (the convex functions r_a are potentially different for each link, but depend only on the load of the corresponding link a.). Flows are unsplittable as $x_k^p \in \{0,1\}$; the problem can be relaxed to splittable flows by allowing $x_k^p \in \mathbb{R}^+$.

Splittable multi-commodity flow problems with non-linear convex costs is a well-known problem, for which first methods have been introduced by [2]. A more recent survey [3] provides an overview of several efficient approach to this problem.

However, these methods are solely consider in the splittable flows context, and don't necessarily apply efficiently to solve unsplittable multi-commodity flow problems.

3 Proposed Approach

In this presentation, we will propose a Mixed Integer Linear Program where an exponential number of variables are added to consider the convex function. These new variables represent new pieces of these approximations. In this model, we keep the path variables presented previously ; both sets of variables are generated with a column generation scheme.

This approach has the advantage of solving the non-linear multicommodity flow problem with a column generation approach, which ensures capacities are respected, and allows directly plug it into a Branch-and-Price algorithm and efficiently solving the (integer) unsplittable multi-commodity flow.

Most known efficient approaches to solve the multi-commodity flow with non-linear convex costs (like for instance the ACCPM method described in [1]) would not share this capability. The proposed approach also allows a large level of flexibility for adding new constraints into the problem.

Références

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