Finding several solutions to planning problems with specific time windows structure through quantum walk and quantum amplitude amplification

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1 On the use of quantum computing

Classical operations research algorithms primarily focus on identifying the best (and ideally optimal) solution for a given problem. In quantum computing, many applications strive to surpass the performance of classical algorithms in finding such solutions. However, the existing noise in quantum processors currently available [1] and the intrinsic probabilistic nature of quantum processes cannot guarantee attaining the desired solution on a real quantum computer. Additionally, from an industrial standpoint, it is often impractical to account for all real constraints associated with a given problem even on classical models. The 'optimal solution' derived from models without these constraints becomes impractical for use in many cases requiring the application of repair heuristics or expert knowledge. Hence, it is sometimes more interresting to transform the optimisation problem into a constraint satisfaction (CSP) one where the objective would be to obtain a set of solutions with different structures and a given minimum quality to feed the post-processing algorithms. In contrast to classical algorithms, which are typically not designed to generate this kind of set, quantum algorithms leverage the superposition of states and may prove efficient for such tasks. In applications where repair heuristics are required, the presence of noise could becomes less critical as long as we can obtain several solutions or partial solutions of good quality. The inherent probabilistic nature of quantum algorithms leads to a naturally diverse range of solutions. This presentation introduces a quantum algorithm designed for this purpose, we address a job scheduling problem with unitary duration, resource constraints and specific time windows structure based on real case study at EDF.

2 Use case

Consider a set of machines $i \in I$ and for each machine i a set of job K_i to schedule on this machine. Each job $(i, k) \in K_i$ has a unitary duration and a time window $[E_{ik}, D_{ik}]$. Time windows follow a given pattern : time window of job (i, k) only intersect with time windows of the same index k from other units and has a range of size $3k + 1$. The problem has two additionnal constraints :

— Minimum and maximum time spacing between consecutive jobs of an unit.

— Renewable resource constraints limiting the number of simultaneous jobs.

Given an initial set of dates (X_{ik}^{init}) which doesn't correspond to a feasible solution, the Job-Shop Reparation Constraint Satisfaction Problem (JSR-CSP) aims at finding feasible schedules within a specific 2-norm distance from (X_{ik}^{init}).

This problem encapsulates a refined model tailored for multi-year planning in power plant maintenance. In this context, machines represent production units, and jobs correspond to maintenance outages of specific units. The inclusion of spacing constraints and the structured time windows aligns with the physical constraints inherent in maintenance operations.

3 Using quantum walk and amplitude amplification algorithms

Quantum Walk [2] places a quantum system in a superposition of states, enabling the simultaneous exploration of the entire search space. Amplitude Amplification algorithms [3], an extension of the renowned Grover Search algorithm [4], leverage quantum superposition to accelerate searches for marked elements in the solution space. In our methodology, we associate each conceivable combination of dates for a unit with a combination of qubits. We employ quantum walk to generate a superposition of feasible schedules. This process respects time windows and constraints related to intervals between consecutive outages. Subsequently, two oracles come into play : one addressing resource constraints and the other handling deviations from the initial schedule. These oracles mark feasible schedules within a specified distance to the initial solution, and we finally apply a fixed-point amplitude amplification, implemented using quantum singular value transformation (QSVT) [5]. The procedural flow of our algorithm is depicted in Figure 1.

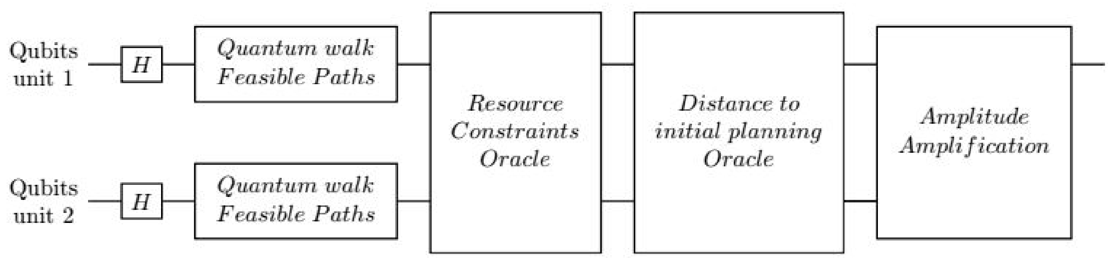

FIG. 1 – Proposed algorithm

We present a thorough analysis of the classical complexity associated with this use case and performed preliminary experiments in simulation. This serves as a foundational proof of concept for our approach. Our evaluation involves a comparison between the quantum algorithm and classical algorithms, focusing on the diversity of the solutions achieved.

Références

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