Multi-surrogate Radial Basis Function Assisted Differential Evolution for Multi-server Congested p-median Problem

Muhammad Sulaman[1], Mahmoud Golabi[1], Mokhtar Essaid[1], Mathieu Brévilliers[1], Julien Lepagnot[1], Lhassane Idoumghar[1]

University of Haute-Alsace, IRIMAS UR 7499, F-68100 Mulhouse, France

{firstname.lastname}@email.adresse

Mots-clés : *multi-server congested facility location problem, radial basis function, differential evolution, surrogate-modeling, p-median facility location problem*

1 Introduction

The Facility Location Problem (FLP) [1] is a widely researched topic in operations research, with applications ranging from optimal warehouse placement to coastal search and rescue stations. It includes deterministic and stochastic variations, with the latter incorporating uncertain parameters such as demand or cost [4]. The introduction of congestion by [2] led to the study of Congested Facility Location Problems (CFLPs).

FLP is categorized into center, covering, and median problems based on the objective function. Center problems minimize maximum distances, covering problems maximize customer coverage, and median problems minimize total travel or waiting times. The NP-hardness of the p-median problem on a general graph was established by [3], prompting the use of metaheuristic algorithms such as genetic algorithms, simulated annealing, and variable neighborhood search.

This study extends the research on the p-median problem to the Multi-Server Congested Facility Location Problem (MSCFLP), aiming to develop an efficient solution approach. In addition to FLP, the MSCFLP introduces additional factors, such as multi-servers and congestion, addressing practical challenges. The objective is to minimize overall expected transportation time and cumulative waiting times.

The complexity of median problems and constrained non-linear mixed integer programming models, known for being NP-hard, is magnified in time-sensitive and dynamic scenarios such as post-disaster emergency facility locations. Multi-server and congestion add up the complexity and computational overhead to the problem. To address computational overhead in solving MSCFLPs, especially in time-sensitive scenarios, this study introduces a novel multi-surrogate-assisted method RBFN-DE. The proposed method leverage machine learning, specifically the radial basis function (RBFN) [5], combined with differential evolution (DE) [6], to enhances decision-making in facility location and resource allocation, improving efficiency and, providing robust and efficient solutions for the MSCFLP.

2 Proposed Method

The proposed algorithm utilizes the DE as the primary optimizer. In each generation, three radial basis function-based surrogate models, $M1$, $M2$, and $M3$, are reconstructed and updated. The algorithm starts with an initial population through Latin hypercube sampling with objective values calculated by an actual objective function. This population is incorporated into the training dataset T. Prior to entering the main loop, three RBFN models ($M1$, $M2$, and

$M3$) are established. The training dataset T is subsequently divided into three equal subsets ($T1$, $T2$, and $T3$), each utilized to train the corresponding surrogate model M_i.

At the onset of each generation, all three models are updated using pseudo-fitness values. Pseudo-fitness is determined by averaging the fitness values of the top individuals predicted by a combination of two models. For instance, $f^1_{avg} = (f_1 + f_2)/2$, where f_i represents the predicted fitness of the top individual of $M1$, and f_2 is the predicted fitness value of model M_2. Similarly, $f^2_{avg} = (f_2 + f_3)/2$ and $f^3_{avg} = (f_3 + f_1)/2$ are calculated. These pseudo-fitness values (f^1_{avg}, f^2_{avg}, f^3_{avg}) are then employed to update $M1$, $M2$, and $M3$, respectively.

Following the model updates, DE-based crossover and mutation operators are applied to generate an offspring population. For the first two generations, the entire population is evaluated using the actual objective function and added to the training dataset T. From the third generation onwards, only 10 percent of the newly generated offspring population is evaluated by the actual function, and the remaining 90 percent is predicted by each model ($M1$, $M2$, and $M3$). The predicted populations are sorted, and the top individuals are selected for the subsequent generation. Importantly, the size of T increases in each generation.

At the end of each generation, T is shuffled and divided into $T1$, $T2$, and $T3$, which are then used to reset each $M1$, $M2$, and $M3$, respectively. This process continues through subsequent generations untill the stopping criteria is met, ensuring dynamic model adaptation and effective exploration-exploitation balance.

3 Experimental Results

The parameters of DE are tuned using HalvingRandomSearchCV initially, followed by HalvingGridSearchCV from scikit-learn for optimal configurations. To address imprecise demand data in p-median problems, we created eight benchmark test instances (Node500 to Node3000). These instances assess the performance optimization algorithms, with sizes scaling based on node count. For instance, Node500 has 500 nodes, while Node3000 has 3000 nodes.

The experimental results demonstrated that DE obtained lower standard deviation, indicating better stability in terms of objective values while, RBFN-DE maintained acceptable stability with reliable and diverse solution capabilities. Both algorithms effectively explored the solution space, with RBFN-DE showing computational efficiency improvements (24% to 44%) compared to DE.

Statistical tests, including ANOVA, Kruskal-Wallis, and Mann-Whitney U, found no significant difference in objective values among algorithms. However, RBFN-DE significantly outperformed in terms of computational time. Overall, while DE slightly excelled in objectives, RFDE stood out in computational efficiency, making it ideal for time-sensitive optimization tasks.

Références

[1] Arabani, A. B., & Farahani, R. Z. (2012). Facility location dynamics : An overview of classifications and applications. Computers & Industrial Engineering, 62(1), 408-420.

[2] Larson, R. C. (1974). A hypercube queuing model for facility location and redistricting in urban emergency services. Computers & Operations Research, 1(1), 67-95.

[3] Kariv, O., & Hakimi, S. L. (1979). An algorithmic approach to network location problems. I : The p-centers. SIAM journal on applied mathematics, 37(3), 513-538.

[4] Bieniek, M. (2015). A note on the facility location problem with stochastic demands. Omega, 55, 53-60.

[5] Du, K -L., and M. N. S. Swamy. "Radial basis function networks." Neural networks in a softcomputing framework (2006) : 251-294.

[6] Storn, R., & Price, K. (1997). Differential evolution-a simple and efficient heuristic for global optimization over continuous spaces. Journal of global optimization, 11(4), 341.