Benders Decomposition assissted by neutral atoms - a PoC

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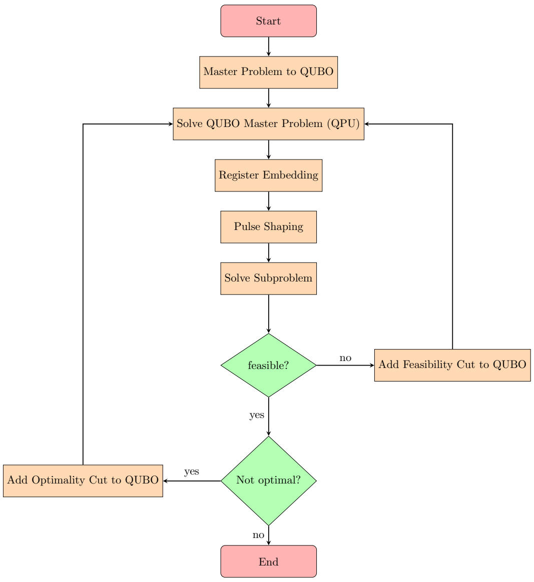
1 Introduction

Mixed Integer Linear Programming (MILP) is crucial for solving a variety of real-world optimization problems. A widely used method for tackling MILP problems is Benders Decomposition (BD). It involves iteratively addressing the master problem (MP), which manages integer variables, and the subproblem (SP), which typically deals with continuous variables. While BD is efficient in many applications, it faces challenges like slow convergence and weak solutions, leading to the development of improved stabilization methods and strategies for generating multiple solutions. The emergence of hybrid quantum algorithms, which merge classical and quantum computing, presents new avenues for solving MILPs using BD [1], potentially overcoming some of its limitations. These algorithms are especially suitable at handling the MP, that can be reformulated as a Quadratic Unconstrained Binary Optimization (QUBO), a well-suited model for quantum plateforms. One quantum computing framework that has proven efficiency in solving optimization problems is quantum computing using neutral atoms [2], wherein atoms act as qubits manipulated by lasers. Extending our prior research that applied neutral atom Quantum Processing Units (QPUs) for a hybrid column generation algorithm to solve the ILP graph coloring problem [3], our current study expands to MILPs. We introduce a hybrid classical-quantum Benders decomposition method, utilizing, for the first time, neutral atom computations. This approach leverages the computational strength of neutral atoms and presents a unique solution to the traditional challenges in solving the master problem in BD. This abstract paper provides an overview of the fundamental principles of our approach and presents some preliminary results, demonstrating the potential efficiency of our hybrid algorithm.

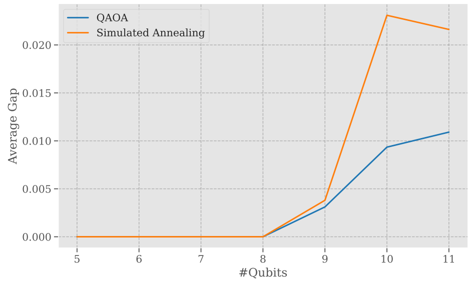
2 Methodology and results

The hybrid classical-quantum Benders decomposition approach is illustrated in Figure 1a. The MILP is decomposed into two problems : a Master Problem (MP) and a SubProblem (SP). The MP is processed on a neutral atoms based QPU, while the SP is handled by a classical CPU using CPLEX. The process involves converting the MILP master problem into a QUBO, suitable for the quantum computing environment. The QPU utilizes laser-manipulated atoms, aligning with the system's Hamiltonian to manage qubit dynamics and interactions. The efficiency of this approach relies on well-designed algorithms, particularly focusing on 'register embedding' to arrange atoms in a way that mirrors the QUBO, and 'pulse shaping' to optimize the laser for effective problem-solving. Research in Quantum Approximate Optimization Algorithms (QAOA) has shown promising outcomes, indicating that precise pulse shaping and embedding can yield robust solutions in quantum optimization.

we develop a heuristic for efficient atom placement in register embedding respecting the spacial constraints of the QPU. In addition, we implement a QAOA algorithm that incorporates gradient descent techniques for pulse shaping. We also provide a Proof of Concept (PoC) to demonstrate the practical application of our solver. We conducted experiments on 1000 randomly generated small-sized MILPs, observing that our method outperforms simulated annealing (SA). It achieved a higher feasibility rate of 95% compared to only 45% with simulated annealing, and required fewer iterations. As seen in Figure 1b, our approach also demonstrated a better optimality gap, indicating its superior solution quality when compared to SA.



(a) Hybrid quantum-classical Benders decomposition algorithm



(b) Average gap per number of qubits

FIG. 1 – The hybrid BD algorithm and preliminary results.

3 Conclusion and perspectives

In this study, we have developed a novel method for solving Mixed Integer Linear Programming (MILP) problems by using Benders' decomposition with quantum computing neutral atom devices. Our approach, validated against classical heuristics, offers a promising direction in quantum computational

Future developments include a qubit resource management strategy, limiting qubit numbers while refining penalty terms to improve solution quality. Another interesting direction is accelerating algorithm convergence by incorporating multiple feasibility and optimality cuts.

Références

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