Uncertainty reduction in static robust optimization

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1 Introduction

In this work, we are interested in robust optimization problems of the form:

$$\begin{aligned} \min_{y\in Y} \quad & f^\top y && \text{(Static-Robust)} \\ \text{s.t.} \quad & H(\xi)y \leq g && \forall \xi \in \Xi \end{aligned}$$

where the set $Y \subseteq \mathbb{R}^n$ defines the deterministic structure of solutions y and may incorporate integrality restrictions, $\Xi \subseteq \mathbb{R}^q$ is a polytope, $H(\xi)$ for $\xi \in \Xi$, f and g are real matrices and real vectors of conforming dimensions, respectively. We assume that, all uncertain parameters are affine functions of $\xi \in \Xi$. Problem (Static-Robust) is a well-studied problem in the literature (see [1]) and can be numerically treated using well-known reformulation techniques. However, this model cannot handle situations in which the decision-maker can affect the uncertainty set. One such situation is *uncertainty reduction* where the decision-maker can undertake proactive actions in order to reduce the range of uncertain parameters expressed by the decision-dependent uncertainty set:

$$\Xi(x) = \{\xi \in \mathbb{R}^q_+ \mid D\xi \leq d, \xi \leq v + w \circ (e - x)\}, \tag{1}$$

where $v, w \in \mathbb{R}^q_+$, $x \in X \subseteq \{0,1\}^q$ is a binary decision vector, and e is the vector of all ones. In this work, we study the variant of (Static-Robust) with uncertainty reduction:

$$\begin{aligned} \min_{x\in X\subseteq\{0,1\}^q, y\in Y} \quad & c^\top x + f^\top y && \text{(UR-Robust)} \\ \text{s.t.} \quad & Ax + H(\xi)y \leq g && \forall \xi \in \Xi(x). \end{aligned}$$

We dedicate a particular interest to the min-max combinatorial variant of the above robust problem with binary optimization variables y and only objective uncertainty with $q = n$:

$$\min_{x\in X\subseteq\{0,1\}^q, y\in Y\subseteq\{0,1\}^n} \max_{\xi\in\Xi(x)} c^\top x + (f+\xi)^\top y. \tag{UR-Min-Max}$$

2 Literature review

The first mention of decision-dependent uncertainty sets in the robust optimization literature dates back to [8] where the authors use its expressive power to better model the application at hand, specifically, a software partitioning problem involving multiple instantiations. The notion has also been used by [6, 7], who show how the use of decision-dependent budgets can reduce the conservatism of the so-called budgeted uncertainty set [3], sometimes at no extra computational cost. In yet another context, [4] rely on decision-dependent uncertainty sets to model K-adaptable policies, wherein variables x allow to partition set Ξ optimally. The authors of [5] introduce the uncertainty reduction model (UR-Robust), for which they propose different formulations as well as detailed numerical experiments that illustrate the possible impact of uncertainty reduction. They additionally consider MILP reformulations and a hardness proof for robust optimization problems with a more general decision-dependent uncertainty set structure.

3 Methodological development and results

We first consider the more general model (UR-Robust) for which we propose a new reformulation in the case where $D \geq 0$, which does not involve big-M coefficients whenever y is binary. Our main result shows that when $X = \{0,1\}^q$, solving (UR-Min-Max) amounts to solving $n+1$ deterministic optimization problems in the form $\min_{y \in Y} \tilde{f}^\top y$ in line with the seminal result of [2], and, in particular, that (UR-Min-Max) is polynomially solvable whenever the deterministic problem is for any $\tilde{f} \in \mathbb{R}^n$. We complement that positive result by showing hat (UR-Min-Max) remains NP-Hard when a general set $X \subseteq \{0,1\}^q$ is considered (thereby echoing the results of [5] even when the decision-dependence is restricted to the special case of uncertainty reduction). Finally, we numerically illustrate our theoretical results on the shortest path instances described by [5] and compare it to the reformulations proposed therein.

4 Conclusions et perspectives

The results proposed in this work are a first step in the right direction for incorporating decision-dependent uncertainty sets in robust optimization. These have a wide range of applications but have not yet been methodologically explored in the literature. Future work may consider extending the favorable complexity results to more generic problems as well as algorithmic development for NP-Complete variants.

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