On Solving the Dynamic Robust Master Surgical Schedule under Multiple Uncertainties via a Column-and-Constraint Generation Algorithm

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1 Introduction

This paper aims to efficiently solve the dynamic master surgical schedule (MSS) and surgical case assignment problem (SCAP) under uncertainty using a Column-and-Constraint Generation (C&CG) algorithm [2]. The goal is to minimize assignment cost while considering several operating rooms (ORs) restrictions such as OR resources reservations and OR sessions parallelism, surgeons’ and downstream resources availability, etc. Various uncertainties, such as surgery duration and postoperative length of stay (LOS) in the intensive care unit (ICU) can disrupt schedules. The variability in LOS may lead to surgery cancellations and premature discharges owing to insufficient ICU bed availability. This work addresses both OR and downstream resource constraints, considering surgery duration and LOS uncertainties. We propose a two-stage robust optimization (RO) approach and employ a C&CG algorithm to solve the MSS and the SCAP.

2 Column-and-Constraint Generation for the robust MSS

To enhance robustness, we employ the polyhedral uncertainty sets (1) and (2) by [1]. We consider uncertain surgery durations $d_{is} \forall i \in I_s$ (set of patients) $\forall s \in S$ (set of surgical specialty) that falls within the range $[\bar{d}_{is}, \bar{d}_{is} + \hat{d}_{is}]$ and $\lambda_{is}$ is the normalized deviation from the nominal surgery duration. Similarly, we handle uncertain ICU LOS $l_{ICU}^{is} \forall i \in I_s \forall s \in S$ within $[\bar{l}_{ICU}^{is}, \bar{l}_{ICU}^{is} + \hat{l}_{ICU}^{is}]$, and $\eta_{is}$ represents the normalized deviation from the nominal ICU. We introduce $\Gamma_d$ and $\Gamma_l$ as robustness budgets, limiting the number of variables at their extreme values.

$$\Xi_{d}^{rj} = \left\{ d_{is} \in \mathbb{R}^n \mid d_{is} = \bar{d}_{is} + \hat{d}_{is} \lambda_{is}, \sum_{s \in S} \sum_{i \in I_s} \lambda_{is} \leq \Gamma_d, 0 \leq \lambda_{is} \leq 1 \right\}$$

$$\Xi_{ICU}^{rj} = \left\{ l_{ICU}^{is} \in \mathbb{R}^n \mid l_{ICU}^{is} = \bar{l}_{ICU}^{is} + \hat{l}_{ICU}^{is} \eta_{is}, \sum_{s \in S} \sum_{i \in I_s} \eta_{is} \leq \Gamma_l, 0 \leq \eta_{is} \leq 1 \right\}$$

The C&CG algorithm 1 efficiently handles a two-stage resolution process for the RO model. $\chi_{isrj}$ is a decision variable set to 1 if surgery $i \in I_s$ is assigned to day $j \in J$ (set of days) in room $r \in R$ (set of ORs) and 0 otherwise. $O^{max}$ is the capacity of the OR session. $r_{is}$ is 1 if patient $i \in I_s$ requires an ICU bed and 0 otherwise. $o_{rj}$ is a decision variable capturing the overtime in the OR session on day $j \in J$ and room $r \in R$, and $z_j$ is a decision variable.
Algorithm 1 C&CG Algorithm for Robust MSS

Initialization:
\[ LB = -\infty, \quad UB = +\infty, \quad K = 0, \quad O = \emptyset \]

Master:
\[ \min \sum \sum_{s \in S} \phi_{is} x_{isrj} + \eta \]
\[ \text{s.t.} \]
\[ \eta \geq \sum \sum_{r \in R, j \in J} c_{rj} \omega_{rj} + \sum_{j \in J} p_j z_j^k \quad \forall k \in O \]
\[ \quad \text{if} \quad c \text{ is overtime cost and } p \text{ is unit cost for not having a ICU bed} \]
\[ \sum \sum_{s \in S} \chi_{isrj} = 1 \quad \forall s \in S \quad \forall i \in I_s \]
\[ \quad \text{if} \quad \text{specialty-to-rooms assignment} \]
\[ \sum_{r \in R, j \in J} \chi_{isrj} = \text{the optional solution is found} \]

Add specialty-to-OR restrictions constraints
Add OR sessions-per-specialty restrictions constraints
Add specialty-to-OR restrictions constraints
Add OR reservation constraints
\[ \sum \sum_{s \in S} \sum_{z \in I_s} \sum_{j \in J} \sum_{j' > j} r_{is} x_{isrj} \leq \nu_j + z_j^k \quad \forall j \in J \quad \forall k \leq K \]
\[ \text{if} \quad UB - LB \leq \epsilon \text{ then} \]
\[ K \leftarrow K + 1, \quad O \leftarrow O \cup \{ K + 1 \} \text{ and go to Master.} \]

else
Add variables \( x_{isrj}^{K+1} \) and \( z_j^{K+1} \) and the following constraints to the Master problem
\[ \eta \geq \sum \sum_{r \in R, j \in J} c_{rj} \omega_{rj}^{K+1} + \sum_{j \in J} p_j z_j^{K+1} \]
\[ \sum \sum_{s \in S} \sum_{z \in I_s} \sum_{j \in J} \sum_{j' > j} r_{is} x_{isrj} \leq \nu_j + z_j^{K+1} \quad \forall j \in J \]
\[ \end{if} \]

where \( d_s^{K+1} \) and \( \nu_{ICU}^{K+1} \) are the optimal scenarios solving the \( S_p^{K+1} \) and \( D_p^{K+1} \)

Update \( K \leftarrow K + 1, O \leftarrow O \cup \{ K + 1 \} \) and go to Master.

References
