On Solving the Dynamic Robust Master Surgical Schedule under Multiple Uncertainties via a Column-and-Constraint Generation Algorithm

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1 Introduction

This paper aims to efficiently solve the dynamic master surgical schedule (MSS) and surgical case assignment problem (SCAP) under uncertainty using a Column-and-Constraint Generation (C&CG) algorithm [2]. The goal is to minimize assignment cost while considering several operating rooms (ORs) restrictions such as OR resources reservations and OR sessions parallelism, surgeons' and downstream resources availability, etc. Various uncertainties, such as surgery duration and postoperative length of stay (LOS) in the intensive care unit (ICU) can disrupt schedules. The variability in LOS may lead to surgery cancellations and premature discharges owing to insufficient ICU bed availability. This work addresses both OR and dowstream resource constraints, considering surgery duration and LOS uncertainties. We propose a two-stage robust optimization (RO) approach and employ a C&CG algorithm to solve the MSS and the SCAP.

2 Column-and-Constraint Generation for the robust MSS

To enhance robustness, we employ the polyhedral uncertainty sets (1) and (2) by [1]. We consider uncertain surgery durations d_{is} $\forall i \in \mathcal{I}_s$ (set of patients) $\forall s \in \mathcal{S}$ (set of surgical specialty) that falls within the range $[\overline{d}_{is}, \overline{d}_{is} + \widehat{d}_{is}]$ and λ_{is} is the normalized deviation from the nominal surgery duration. Similarly, we handle uncertain ICU LOS l_{is}^{ICU} $\forall i \in \mathcal{I}_s$ $\forall s \in \mathcal{S}$ within $[\overline{l_{is}^{ICU}}, \overline{l_{is}^{ICU}} + \widehat{l_{is}^{ICU}}]$, and η_{is} represents the normalized deviation from the nominal ICU. We introduce Γ_d and Γ_l as robustness budgets, limiting the number of variables at their extreme values.

$$\Xi_{rj}^{d} = \left\{ d_{is} \in \mathbb{R}^n | d_{is} = \overline{d}_{is} + \widehat{d}_{is}\lambda_i, \sum_{s\in\mathcal{S}} \sum_{i\in\mathcal{I}_s} \lambda_{is} \leq \Gamma_d, 0 \leq \lambda_{is} \leq 1 \right\} \tag{1}$$

$$\Xi_{j}^{ICU} = \left\{ l_{is}^{ICU} \in \mathbb{R}^n | l_{is}^{ICU} = \overline{l_{is}^{ICU}} + \widehat{l_{is}^{ICU}}\eta_{is}, \sum_{s\in\mathcal{S}} \sum_{i\in\mathcal{I}_s} \eta_{is} \leq \Gamma_l, 0 \leq \eta_{is} \leq 1 \right\} \tag{2}$$

The C&CG algorithm 1 efficiently handles a two-stage resolution process for the RO model. χ_{isrj} is a decision variable set to 1 if surgery $i \in \mathcal{I}_s$ is assigned to day $j \in \mathcal{J}$ (set of days) in room $r \in \mathcal{R}$ (set of ORs) and 0 otherwise. $O^{\max}$ is the capacity of the OR session. r_{is} is 1 if patient $i \in \mathcal{I}_s$ requires an ICU bed and 0 otherwise. o_{rj} is a decision variable capturing the overtime in the OR session on day $j \in \mathcal{J}$ and room $r \in \mathcal{R}$, and z_j is a decision variable

Algorithm 1 C&CG Algorithm for Robust MSS

Initialization:

$LB = -\infty$, $UB = +\infty$, $K = 0$, $O = \emptyset$

Master: Solve the master surgical schedule (Master Problem)

$$\min \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}_s} \phi_{is} \chi_{isrj} + \eta$$

s.t.

$$\eta \geq \sum_{r \in \mathcal{R}} \sum_{j \in \mathcal{J}} c_{rj} o_{rj}^k + \sum_{j \in \mathcal{J}} p_j z_j^k \qquad \forall k \in O$$
▷ c is overtime cost and p is unit cost for not having a ICU bed

$$\sum_{r \in \mathcal{R}} \sum_{j \in \mathcal{J}} \chi_{isrj} = 1 \qquad \forall s \in \mathcal{S} \quad \forall i \in \mathcal{I}_s$$

Add specialty-to-OR restrictions constraints

Add Limits on specialty parallelism constraints

Add OR sessions-per-specialty restrictions constraints

Add OR reservation constraints

$$\sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}_s} d_{is}^k \chi_{isrj} \leq O^{\max} + o_{rj}^k \qquad \forall r \in \mathcal{R} \quad \forall j \in \mathcal{J} \quad \forall k \leq K$$
▷ OR capacity

$$\sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}_s} \sum_{r \in \mathcal{R}} \sum_{\substack{j' \in \mathcal{J} \\ j' > j - l_{is}^{ICU\ k}}}^{j} r_{is} \chi_{isrj'} \leq \nu_j + z_j^k \qquad \forall j \in \mathcal{J} \quad \forall k \leq K$$
▷ Daily beds' availability in the ICU

$\chi_{isrj} \in \{0, 1\} \quad \forall s \in \mathcal{S} \quad \forall i \in \mathcal{I}_s \quad \forall r \in \mathcal{R} \quad \forall j \in \mathcal{J} \quad \forall k \leq K$ ▷ (+other MSS variables)

Obtain the optimal solution $(\chi_{K+1}^*, \eta_{K+1}^*, o^{1*}, ..., o^{K*}, z^{1*}, ..., z^{K*})$. Set $LB = \phi_{is}\chi_{K+1}^* + \eta_{K+1}^*$

Recourse:

Solve the subproblems that tackle uncertainty and get objective values S_{K+1}^* and D_{K+1}^*

Update $UB = min\{UB, \phi_{is}\chi_{K+1}^* + S_{K+1}^* + D_{K+1}^*\}$ ▷ S_{K+1}^* and D_{K+1}^* solved using the strong duality

If $UB - LB \leq \epsilon$ **then**

The optimal solution is found

Else Add-Cut:

Add variables o_{rj}^{K+1} and z_j^{K+1} and the following constraints to the Master problem

$$\eta \geq \sum_{r \in \mathcal{R}} \sum_{j \in \mathcal{J}} c_{rj} o_{rj}^{K+1} + \sum_{j \in \mathcal{J}} p_j z_j^{K+1}$$

$$\sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}_s} d_{is}^{K+1} \chi_{isrj} \leq O^{\max} + o_{rj}^{K+1} \qquad \forall r \in \mathcal{R} \quad \forall j \in \mathcal{J}$$

$$\sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}_s} \sum_{r \in \mathcal{R}} \sum_{\substack{j' \in \mathcal{J} \\ j' > j - l_{is}^{ICU\ K+1}}}^{j} r_{is} \chi_{isrj'} \leq \nu_j + z_j^{K+1} \qquad \forall j \in \mathcal{J}$$

end if

where d_{is}^{K+1} and $l_{is}^{ICU\ K+1}$ are the optimal senarios solving the S_{K+1}^* and D_{K+1}^*

Update $K \leftarrow K + 1, O \leftarrow O \cup \{K + 1\}$ and go to **Master**.

capturing the extra beds required in the ICU on day j. The MSS has other decision variables related to specialty-to-rooms and rooms-to-surgeons assignment. In the first stage, it begins by considering only decision variables and constraints in the master problem, gradually reintegrating released components. The second stage aims to identify the worst-case scenario within a polyhedral uncertainty set. By iterating through these operations, more scenarios are included in the master problem, leading to the growth of variables and constraints. This iterative process improves both upper and lower bounds until convergence is achieved. The computational experience is based on real data. Results will be presented at the conference.

References

[1] Bertsimas, D. and Sim, M. The price of robustness, *Operations Research*, 52(1):35–53, 2004.

[2] Zeng, B. and Zhao, L. Solving two-stage robust optimization problems using a column-and-constraint generation method, *Operations Research Letters*, 41(5):457-461, 2013.