An ameliorated model of flexible break constraints for Multi-Activity Shift Scheduling problems

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1 Introduction

The Multi-Activity Shift Scheduling problem (MASSP) is a staff scheduling problem that aims at constructing an optimal working plan to satisfy workload demands [**3**] while respecting several regular constraints. It is a complex optimization problem that is widely arise in various industries such as restaurant, health care [**4**] or client services. As many other staff scheduling problems, the MASSP usually deals with several legal constraints like daily and weekly rest time, daily working amplitude, the number of working days, etc. In this research, we study a MASSP in the restaurant industry with flexible break constraints.

A break is an interruption with fix or flexible duration between two working shifts of an employee. It is one of the most common and important legal constraints in scheduling problems. This constraint assures employees' breaks time during the working day. The breaks are defined by a frame of their starting and ending times as well as their duration. Many studies in the literature have included break constraint criterion into their models [**1, 2**].

In this research, we propose a new mathematical formulation for the MASSP with flexible break constraints where each workload demand has a fixed starting and ending times, and a break must be applied between two separated shifts in a working day. In the new formulation, a series of possible break starting and ending times is used to model them.

Experiments on real instances demonstrate that the proposed formulation reduces the number of variables and constraints, consequently leading to a reduction in the computational time required for solving the studied MASSP compared to existing formulations in the literature.

2 The proposed formulation

In the studied MASSP, a set D of n multi-activity workload demands (or tasks) is to be assigned to a set E of employees. Each demand is an activity with a fixed starting and ending times. If demand j is assigned to employee i, the employee is occupied from the starting to the ending of the demand. A shift is a series of consecutive working tasks involving one or multiple activities. If any employee works on two shifts the same day, they must have a break in-between. Breaks starting and ending times are not fixed and not identical for everyone but are flexible according to the workload demands with a minimum and maximum of duration. The rest time between two working days is not considered a break. Therefore, to reduce the number of decision variables, we define a series of possible break starting times B^s and ending times B^e. To define the schedule of an employee, we use the following binary variables :

— $x_{i,j}$: 1 if task j is assigned to employee i, 0 otherwise.

— $y_{i,t}$: 1 if employee i has a break starting at $t \in B^s$, 0 otherwise.

— $z_{i,t}$: 1 if employee i has a break ending at $t \in B^e$, 0 otherwise.

To respect the maximum length of this abstract, we present below only the constraints related to the flexible breaks. The complete model will be presented in the conference.

$$Minimize: \quad n - \sum_{\substack{i \in E \\ j \in D}} x_{ij} \tag{1}$$

Subject to :

$$\forall i \in E, \forall j \in D, \forall k \in F^j : \quad x_{ik} \leq 1 - x_{ij} \tag{2}$$

$$\forall i \in E, \forall t \in B^s : \quad \sum_{\substack{j \in D \\ j.Start = t}} x_{ij} \leq 1 - y_{it} \tag{3}$$

$$\forall i \in E, \forall t \in B^s : \quad \sum_{\substack{j \in D \\ j.End = t-1}} x_{ij} \geq y_{it} \tag{4}$$

$$\forall i \in E, \forall t \in B^e : \quad \sum_{\substack{j \in D \\ j.End = t}} x_{ij} \leq 1 - z_{it} \tag{5}$$

$$\forall i \in E, \forall t \in B^e : \quad \sum_{\substack{j \in D \\ j.Start = t+1}} x_{ij} \geq z_{it} \tag{6}$$

Let F^j be the set of demands that overlap with demand j. The objective function (1) is designed to minimize the number of unassigned tasks. The constraint (2) ensures that overlapping tasks are not assigned to the same employee. Additionally, constraints (3) and (4) define the starting time of breaks, while constraints (5) and (6) specify their ending time.

3 Experiments and conclusions

We tested the proposed model on several real industrial instances, and the experimental results were compared with those of the reference model [**1**]. Both models were solved using Cplex 20.1.

The experimental results show that the proposed formulation provides improved solutions in terms of plan balancing in most cases. Especially when a resource has low availability in their timetable. As a consequence, the objective value related to benchmark equity is very high, and the workload sharing of other resources is also not optimal. On the other hand, the proposed formulation considers the working time of every resource in its objective function. Thus, even when a resource's performance is low, the others still get a balanced planning. The test results also show that the proposed formulation, in general, had a lower performance in computational time compared to the benchmark formulation.

According to the experiments, we notice that the proposed formulation has advantages in modeling complex break time constraints such as flexible duration, starting and ending intervals, number of breaks per day or week, etc. As a perspective for future work, the proposed model can be applied in heuristic methods to evaluate the breaks duration equity of the MASSP.

Références

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