Matheuristic using machine learning for the Multi-vehicle Inventory Routing Problem

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1 Introduction

Vehicle routing and inventory management problems are closely linked, as a solution to one problem has a major impact on the decisions on the other. When considered together, we have the Inventory Routing Problem (IRP). The IRP aims to define how much and when to deliver to each customer and how to combine customers into routes so that the demands of customers are met over the time horizon at a minimum total cost composed of inventory holding costs and transportation costs. In addition, each customer has a maximum inventory level allowed, and the capacity of vehicles must be respected. A quantity is made available at the supplier each period, and the initial inventory level for all customers and suppliers is known. This work addresses a variant of the IRP considering a set of homogeneous vehicles known as the Multi-Vehicle IRP (MIRP).

The works in the literature on IRP generally consider relatively short time horizons of, on average, five periods. In practice, one period corresponds to one day, which gives a solution for one week. However, for a longer time horizon, we can imagine that routes may be repetitive despite differences in the delivery quantities when visiting the same customer. Based on this assumption, considering clusters of customers and restricting that customers are present on the same route only if they belong to the same cluster would speed up the problem resolution without, possibly, considerably increasing the cost of the best solution obtained.

To analyze the impact of clustering on the quality of the solution obtained, we propose to compare the resolution of the MIRP by two methods. In both methods, we consider a Mixed Integer Linear Programming (MILP) formulation of the MIRP. In the first method, we solve the model using a linear programming solver. In the second method, we pre-define the clusters of customers and then restrict the routing arcs to a subset of the original arcs according to the clusters. Customers are clustered using the K-means machine learning method. Below, we present the main ideas of the mathematical model and briefly explain the clustering method used. The results obtained are compared with those presented in [3].

2 Mathematical formulation and clustering method

The proposed MILP formulation without considering clusters for customers is largely based on the basic flow formulation presented by [1]. Given the set of customers N, where $N' = N \cup \{0\}$ and 0 is the supplier, and the set of time periods T, with $T' = T \cup \{0\}$, the model considers the binary variables $x_{i,j}$ that take value 1 if $i \in N'$ is visited before $j \in N'$, with $i \neq j$, and 0 otherwise. Also, it contains the continuous variables q_i^t and I_i^t indicating, respectively, the quantity delivered to customer $i \in N$ at period $t \in T$, and the inventory level of $i \in N'$ at the end of period $t \in T'$. Lastly, the model considers the continuous variables $a_{i,j}$ representing the load carried by the vehicle from $i \in N'$ to $j \in N'$, where $i \neq j$.

We propose limiting the solution space of the formulation by clustering the customers. To do this, we used the unsupervised K-Means algorithm to divide a set of points (customers in our case) into k homogeneous clusters. Therefore, the number of clusters is a hyperparameter of the method and must be predefined. The first step of the K-Means algorithm is to randomly define k centroids and associate them with k labels. Then, for each point, we associate the point with the closest centroid and its corresponding label. This procedure repeats until the new centroids no longer move from the previous ones. In the proposed method, customers are clustered using their coordinates as features. In the following, we refer to the complete model as simply "MILP" and to the model restricted to clusters of customers as "MILP-clusters".

Clustering considerably reduces the number of arcs in the problem and, consequently, the number of variables and constraints in the model. However, such restrictions can impact the cost of the solution obtained, which is discussed in the preliminary results presented below.

3 Preliminary results and conclusions

The proposed formulation was implemented using the C++ programming language and solved using the CPLEX 12.10.0 solver. The K-Means algorithm was implemented using the Python language and solved using the scikit-learn library. We present below a summary of the results obtained for each set of "low cost" instances with a time horizon of 6 periods proposed by [2], considering a set of 2 vehicles and a number of clusters k equal to 2 and 3. We compared the results obtained using "MILP-clusters" with those found in the literature and with those obtained by "MILP", respectively. We present the gap between the best Upper Bound (UB) provided using "MILP-clusters" and the UB we compare to, and the gap in the running time of both methods. A negative gap means we are better both concerning UB and execution time.

TAB. 1: Comparison of the results obtained by "MILP-clusters"

	Literature				**MILP**			
	Gap$_{\mathbf{UB}}$		**Gap$_{\mathbf{time}}$**		**Gap$_{\mathbf{UB}}$**		**Gap$_{\mathbf{time}}$**	
$\|N\|$	$k=2$	$k=3$	$k=2$	$k=3$	$k=2$	$k=3$	$k=2$	$k=3$
5	**0.11**	0.16	0.39	**-0.90**	**0.11**	0.16	-0.76	**-0.98**
10	**0.07**	0.17	-0.77	**-0.88**	**0.07**	0.17	-0.94	**-0.97**
15	**0.08**	0.22	3.46	**-0.15**	**0.08**	0.22	-0.31	**-0.87**
20	**0.09**	0.18	-0.65	**-0.75**	**0.07**	0.17	-0.16	**-0.39**
25	**0.06**	0.15	-0.81	**-0.84**	**0.05**	0.15	0.00	**-0.16**
30	**0.11**	0.17	-0.90	-0.90	**0.09**	0.14	0.00	0.00

When we use k equal to 2 for "MILP-clusters", we considerably reduce the execution time compared to the results obtained by "MILP" and existing in the literature for most instances. However, we have an average increase in the cost of 8% for "MILP-clusters". When we use 3 clusters, the reduction in average execution time is greater, as can be seen in bold in the Gap$_{\text{time}}$ columns. However, on average, we have an increase in the solution cost of 17%. Better solutions in terms of UB are found by "MILP-clusters" when considering k equal to 2 compared to when k is equal to 3, as highlighted in bold in the columns Gap$_{\text{UB}}$. Therefore, we conclude that solving methods using clustering present satisfactory results in terms of running time and an acceptable impact on cost. Such methods can be used to generate initial solutions.

In future works, we intend to fine-tune the use of clustering in solving the MIRP as a constructive heuristic and combine this method with a local search to improve the quality of the solutions obtained.

References

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