Ensuring Equitable Controller Placement: A Probabilistic Optimization Framework

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1 Introduction

In SDN, the placement of controllers is crucial for network resilience. Our Equitable Controller Location Problem (ECLP) [1, 2, 6] addresses the vulnerability of SDN to cyber-attacks, specifically focusing on maximizing switch coverage during an attack. Unlike conventional approaches, our method introduces a probabilistic model, combining attack uncertainty, robust optimization, and fairness considerations. Building on prior research, we formulate ECLP mathematically using graph representation, random variables, and historical attack data. This approach ensures equitable protection against potential attack nodes, providing a strategic and resilient solution to SDN controller placement.

The method for equitable controller placement in a network vulnerable to cyber-attacks is formulated as a binary optimization problem. Binary variables x_i represent the presence or absence of controllers at candidate locations $i \in N$, and $q_j(x)$ signifies the probability that node j is not monitored. Utilizing the formulation from [6], the probability $q_j(x)$ is expressed as a product of Bernoulli distributions, defined as:

$$q_j(x) = \prod_{i \in N} (1 - p_{ij})^{x_i},$$

where p_{ij} is the probability that node i monitors node j, and a_{ij} is a binary variable indicating the connection between nodes i and j.

We initiate our discourse by establishing a formal definition of equity and elucidating its correlation with lexicographic optimization, as highlighted in the works [3, 5]. The overarching goal is to maximize the minimum protection level across all nodes. The lexicographic approach introduces a vector γ representing the minimum protection level for each node. The optimization problem is then formulated as:

$$\max \quad \gamma \quad s.t. \quad q_j(x) \leq \gamma_j \quad \forall j \in M, \quad \sum_{i \in N} x_i = K, \quad x_i \in \{0, 1\} \quad \forall i \in N.$$

To handle non-linearity, a logarithmic transformation is applied:

$$\log(q_j(x)) = \sum_{i \in N} (\log(1 - p_{ij})) x_i.$$

This transforms the problem into a linear system:

$$\max \quad \gamma \quad s.t. \quad \sum_{i \in N} (\log(1 - p_{ij})) x_i \leq \gamma_j \quad \forall j \in M, \quad \sum_{i \in N} x_i = K, \quad x_i \in \{0, 1\} \quad \forall i \in N.$$

The leximax minimal vector γ is determined using methods from [2, 4].

The second objective, proportional fairness, minimizes the sum of logarithms of protection levels:

$$\sum_{j\in M} \log(q_j(x)).$$

This problem is efficiently solvable:

$$\min \quad \sum_{j\in M}\sum_{i\in N}(\log(1-p_{ij}))x_i \quad s.t. \quad \sum_{i\in N} x_i = K, x_i \in \{0,1\} \quad \forall i \in N.$$

The linearization simplifies the optimization, providing an effective method for achieving equitable controller placement in the network.

2 Conclusions

In this presentation, we address the challenge of optimizing the placement of a limited number M of network controllers to enhance network resilience against potential attacks. We argue for a focus on equitable access of network nodes to controllers, proposing a probabilistic model based on attack reachability probabilities.

Our approach involves estimating probabilities by analyzing a representative set of attacks, allowing us to formulate the controller placement problem as a variant of the Maximal Coverage Location Problem. We introduce both lexicographic and proportional fairness criteria, showcasing efficient solutions for each.

In particular, our Proportionally Fair model proves to be highly efficient, providing equitable controller placement for networks of any size. Evaluation on medium and large size networks reveals that the equitable models deliver competitive results, with the Proportionally Fair model exhibiting a seemingly better worst-case coverage. These findings support the advocacy for a probabilistic approach to modeling attacks and controller placement, endorsing the efficiency and competitiveness of the proposed Proportionally Fair model.

References

[1] T. Ibaraki and N. Katoh, Resource Allocation Problems: Algorithmic Approaches. Cambridge, MA, USA: MIT Press, 1988.

[2] H. Luss, "Equitable Resource Allocation: Models, Algorithms and Applications," John Wiley & Sons, Hoboken, NJ, USA, 2012.

[3] D. Nace and M. Pioro, “Max-min fairness and its applications to routing and load-balancing in communication networks: A tutorial,”IEEE Communications Surveys Tutorials, vol. 10, no. 4, pp. 5–17, 2008.

[4] W. Ogryczak and T. Sliwinski, "On solving linear programs with the ordered weighted averaging objective," European Journal of Operational Research, vol. 148, no. 1, pp. 80-91, 2003.

[5] W. Ogryczak, H. Luss, M. Pioro, D. Nace, and A. Tomaszewski, "Fair optimization and networks: A survey," Journal of Applied Mathematics, vol. 2014, 2014.

[6] M. C. Santos, H. Luss, D. Nace, and M. Poss, "Proportional and maxmin fairness for the sensor location problem with chance constraints," *Discrete Applied Mathematics*, 2019, doi: https://doi.org/10.1016/j.dam.2019.03.004.