Dantzig-Wolfe decomposition for robust balancing of a simple assembly line under uncertainty

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1 Problem

This work introduces a new upper bounding approach, based on Dantzig-Wolfe decomposition and column generation, for the problem of balancing a simple assembly line under uncertainty in task processing times, seeking to maximize an indicator expressing its robustness. The problem involves task precedence constraints, a fixed cycle time, and a fixed number of workstations. This NP-hard optimization problem aims to assign a given set of assembly tasks to workstations so as to find the most robust line configuration satisfying the mentioned constraints. The robustness of the configuration is measured through the concept of *stability radius* with respect to its feasibility [2], *i.e.*, as the maximum increase in task processing times, for which the cycle time constraint remains satisfied.

The balancing problem is usually related to the early stages of the assembly line design process. Therefore, task processing times are often based on estimates or nominal values, as mentioned in [2]. These values can change during line operation, and any deviation from them can deteriorate the assembly line throughput if the cycle time is overrun. This can result in significant economic losses. The main factors that can lead to deviations in task processing times are operator skill level, fatigue and motivation; changes in product specifications and workstation characteristics; and general delays in task execution. In this sense, it is desirable to achieve a task assignment to workstations that protects the corresponding line configuration and its throughput as much as possible from the uncertainty of task processing times.

This specific problem was already discussed in [5]. Named SALBP-S, it was proven to be NP-hard in the strong sense in [5], where the authors proposed a *compact* mixed-integer linear programming (MILP) formulation to handle it, *i.e.*, a formulation with a polynomial number of constraints and variables. Based on some combinatorial aspects of SALBP-S and aiming to solve it efficiently, Rossi et al. [5] also provided tight upper bounds for small-size instances, but optimality gaps remained significant for the larger ones.

To further reduce these gaps, we propose in this communication a new upper-bounding procedure, based on Dantzig-Wolfe decomposition, for SALBP-S. A few other works applying Dantzig-Wolfe decomposition can be found in the literature but for related problems [1, 3]. In the present case, the decomposition allowed specific valid inequalities that significantly improve the upper bound quality.

2 Solution approach

The MILP model, described in [5], is reformulated and strong valid inequalities are introduced to reinforce it. The decomposition leads to variables representing task assignment patterns in each workstation.

New upper bounds are obtained while solving the model's linear relaxation by column generation. It can be observed that the formulation of the corresponding pricing sub-problem requires a pre-computed upper bound, and that solving this sub-problem can be computationally demanding. These characteristics motivated the use of pre-processing techniques for computing tight task assignment intervals and efficient initial upper bounds.

The assignment intervals are computed by considering the SALPB-S as a variant of single-machine scheduling problem, as in [6]. Usually referred to as LB_4, this strategy is also applied to compute lower bounds for SALBP-1. According to our experimental results, the availability of tighter intervals has a positive impact on the performance of the column generation algorithm. Concerning the initial upper bounds, we propose a bisection method based on *destructive improvement* bounds [4].

The Dantzig-Wolfe decomposition approach is compared to CPLEX on the compact MILP formulation of the problem. On average, the proposed approach provides better upper bounds for the most challenging instances. In those cases, improvements reached 70%, while for less difficult instances, even though improvements in upper bounds are not achieved, deviations to the best upper bound remain under 3%. For the cases where the optimality gap is small, the column generation converges fast. Moreover, in a few cases, better feasible solutions can be obtained in a heuristic way using the columns generated in the proposed approach.

The numerical experiments indicate that the use of both pre-processing techniques is beneficial for performance. We also show that producing tighter assignment intervals reduces the number of residual binary assignment variables and enhances the algorithms' efficiency in terms of solution quality and computational time. Finally, large improvements in the initial upper bound were obtained with the proposed bisection method, whose computational time remains under one second.

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