Learning to Sparsify Networks in Column Generation: Application to Multi-Commodity Network Flow Problems

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Column Generation (CG) [3] is a well known method suited to solving linear optimisation problems that have an exponential number of variables. CG consists in solving the Restricted Master problem (RMP), defined as the original problem restricted to a subset of variables, and iteratively adding promising variables by solving a pricing problem. The algorithm stops with a certificate of optimality as soon as no such variables exist.

CG is notably used for solving the Multi-Commodity Flow problem (MCNF) [2]. The latter consists, given a capacitated network and a set of commodities, in routing the commodities from their source to their target at minimum cost while complying with network capacity requirements. In the MCNF path-flow formulation [1], a variable represents a path that can be used to route a commodity, and new variables are found by solving a shortest-path problem for each commodity. This can be computationally expensive for large instances. In telecommunication networks MCNFs are repeatedly solved over the same network with commodities varying from instance to instance. We propose to leverage this information to devise an effective heuristic. Our approach relies on training a graph neural network [4] to identify regions of interest of the graph for each commodity, reducing drastically its size and leading to an improved performance of the pricing algorithm. To achieve this we learn to sparsify the graphs on which the pricing problem is performed all the while preserving the optimum (see Figure 1). We evaluate our method on a set of realistic MCNFs, showing a reduction in computation time with only slight degradation of the objective.

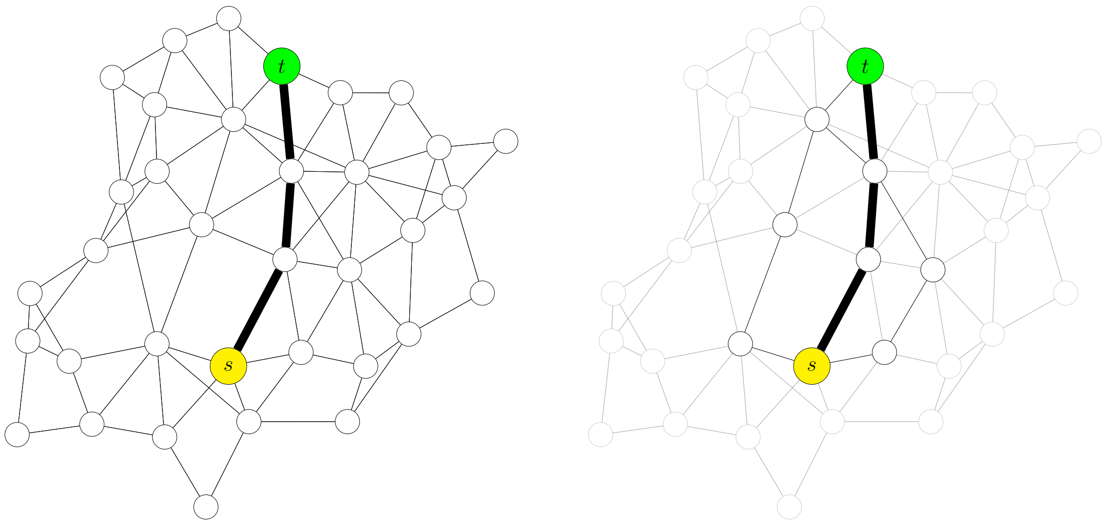

FIG. 1: *left*: original network with a candidate path from s to t , *right*: sparsified graph, the shaded parts of the network are ignored during pricing.

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