Inventory Control for Periodic Intermittent Demand

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1 Introduction

Intermittent demand is characterized by many periods with no demand at all, that are seemingly randomly interspersed with demand occurrences. It is unknown when the next demand will occur, and there is uncertainty about the size of demand when it occurs. This compound nature of demand complicates efficient inventory management, as two sources of uncertainty are present.

Intermittent demand is commonly observed in practice, for example in sectors such as manufacturing, aviation, IT, and in the petrochemical industry [3]. Some companies observe intermittent demand for over half of the products in their inventories [2].

Specific approaches have been developed to tackle uncertainty in the demand. The analysis of intermittent demand times series is commonly decomposed to create separate estimates for the time between demand occurrences (the inter-demand interval) and the size of a demand occurrence, initiated by the work of Croston [1]. In the existing literature, these analyses implicitly assume that the time between demand occurrences is Markovian, i.e, the probability of observing a demand in a certain period is considered to be independent of the time since the last demand occurrence. Data from practice, however, indicate that the times between demand occurrences are often not Markovian. Non Markovian times between demand occurrences lead to periodicity in demand processes that have recently also been observed by [5] and [6].

2 Approach and contributions

This research is the outcome of a joint research project with a company in the chemical industry. The company observes intermittent demand for many of its items, and data suggest periodicity in the demand occurrences. Nonetheless, the company currently does not consider this intermittence in its demand forecasting or inventory decision making. Moreover, it ignores the periodicity in the intermittence of the demand occurrences. Instead, their current practice uses standard techniques for managing their inventories.

In this paper we therefore propose a demand model that accommodates such periodic intermittent demand. We model demand as a discrete compound renewal process. We consider a single stock-point under periodic review. Unsatisfied demand is backordered at a backorder penalty cost per time unit and inventory on-hand at the end of a period incurs a holding cost. We allow for positive lead times. We use a Markov decision process formulation to study the structure of optimal policies.

The main contributions of our paper include descriptions of the form of optimal order policies. In particular, we show that the optimal policy is a state-dependent base-stock policy, where the state is the time since the last demand observation. We prove this by induction on the value function. We also show that there exist state-dependent base-stock policies for which the optimal base-stock level is non-decreasing in the time since the last demand. As such, any algorithm that searches for optimal (or good) base-stock levels can constrain the search space to non-decreasing base-stock levels only. Contrary to what may be expected, this result is not proven by showing submodularity of the value function. Instead we prove sensitivities of optimal base-stock levels with elementary arguments, which can provide new perspectives [4]. We characterize the sensitivities of the base-stock level to the current state that exploits structure in compound renewal processes directly.

Additionally, we contribute to inventory control practice through the development of numerically stable algorithms that can scale to large assortments as encountered in industry. We also benchmark the performance of different approaches on real data sets from our project partner company as well as data sets from several other industries.

3 Results

We benchmark two fast heuristics against the optimal policy in a numerical experiment in which the demand process is assumed to be known. We identify settings where it is especially valuable to exploit the periodicity inherent in compound renewal processes. We show that the optimality gap of both heuristics increases when demand becomes more intermittent, when the coefficient of variation of the time between demands is small, and when the lead time is short. These results show the significant value of correctly using information on demand periodicity and on the time since the last demand occurrence in the order decisions. Furthermore, we observe that even when this information is only included in a myopic way, there are already large gains compared to when a stationary inventory policy is applied.

Second, we evaluate and benchmark the performance when the demand process is not known but demand data is available. We use real data from the chemical company that inspired our research, as well as 4 data sets from the literature. These data sets represent diverse industries in which our model adds value. These industries are (i) specialty chemicals, (ii) air force spare parts, (iii) car parts, and (iv) retail. By benchmarking the performance on different data sets we ensure that our findings can also be generalized to other industries than that of our partner company.

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