Do we care about poll manipulation in political elections?

Vincent Mousseau, Henri Surugue, Anaëlle Wilczynski
MICS - CentraleSupélec - Université Paris Saclay
{vincent.mousseau, henri.surugue, anaelle.wilczynski}@centralesupelec.fr

Mots-clés: computational social choice, game theory, voting theory, manipulation.

1 Introduction

A major topic in computational social choice [3] is the study of political elections. With the rising importance of information in our society, it is becoming increasingly important to consider the power of knowing a relevant information in social decision making. Moreover, we already know since the Gibbard and Satterthwaite’s theorem that we cannot avoid strategic voting with a reasonable voting rule. This fact has led to various approaches to the understanding of political elections, such as iterative voting [4], which considers successive strategic moves from the voters who change their ballot according to the information given to them.

Information is therefore a key element, and grants power to the person who disseminates it. Following the model of Baumeister et al. [1], we consider the constructive problem of control where the polling institute wishes to favor a specific candidate in disseminating information. In particular, we want to study the frequency of successful control in practice using a probabilistic approach to the distributive law of elections [2]. We analyse the two following versions of the problem. First, the unrestricted problem describes the situation where the polling institute can give any score information. Second, the restricted problem refines our hypothesis to make it more realistic: it allows the polling institute to send only score information that would be considered reasonable by the voters, i.e., close to a ground truth that may correspond to the results of a past election, or another poll.

2 Model

Let $N$ be a set of voters where $N = \{1, ..., n\}$, and $M$ be a set of candidates where $M = \{1, ..., m\}$. Each voter $i \in N$ has preferences over candidates represented by a linear order $\succ_i$ over candidates. The winner of the election is determined by the Plurality voting rule where ties are broken lexicographically. More formally, let $b_i \in M$ denote the ballot of voter $i$ and $b \in M^n$ denote the ballot profile. The winner under Plurality of the ballot profile $b$ is $W_P(b) \in \arg\max_{x \in M} s_b(x)$ where $s_b(x) := |\{i \in N : b_i = x\}|$ with a lexicographic tie-breaking if necessary. We will denote $b^T$ the truthful ballot profile, i.e., $b_i^T \succeq_i x$ for every candidate $x \in M$.

In this model, a polling institute sends out a score at the beginning of the process then each voter votes strategically with respect to that information and we discover the winner of the election. Let $I$ be the space of all possible scores that the polling institute can announce: $I := \{s \in \mathbb{N}^m \mid \sum_{j=1}^{m} s_j = n\}$.

First, we want to describe the dynamics of voters’ responses. We say that a candidate $c$ is a potential winner for voter $i$ if when $i$ votes for $c$, then she believes, w.r.t. a given announced score $s \in I$, that $c$ will win the election. We denote $PW_s^i$ the set of potential winners for voter $i$ with respect to announced score $s$. The vote of $i$ may or may not be important for the victory of $c$. When the vote of $i$ matters, we say that $i$ is pivotal for $c$ in announced score $s$. We consider the following best response for each voter $i$ w.r.t. announced score $s$: $i$ deviates
from her current vote to another candidate $c^*$ if she is pivotal for $c^*$ w.r.t. score vector $s$ and $c^*$ is her preferred candidate in $PW_i^s$. Let $b^*$ denote the ballot profile of voters’ best response deviations after the score $s$ has been announced.

Second, we want to describe the behaviour of the polling institute who may have its own interest in the election. Let $x$ be the target candidate of the polling institute, i.e., it wants $x$ to be elected. We analyse the following control problem by the polling institute: does there exist a score $s \in I$ to announce such that $\mathcal{W}_P(b^*) = x$?

We use probabilistic tool to describe how frequently we can expect a manipulation from the polling institute in practice even if in some cases the problem is known to be NP-hard [1]. We use impartial culture as a first approach but we keep in mind that further results may be obtained with other types of distributions over preferences profiles [2]. More precisely, impartial culture will be defined as a uniform distribution on the set $\mathcal{I} = \{\succ_i, i \in N\}$.

3 Probability of successful poll control

3.1 Unrestricted problem

First, we describe the situation where the polling institute is allowed to send any score. Naturally, the probability of control under impartial culture is very high. Furthermore, we prove that it asymptotically converges to 1 as the election becomes larger in terms of voters.

3.2 Restricted problem

Second, we extend our analysis to the case where the polling institute is only allowed to give a score which is close to the ground truth (i.e., the truthful score), with respect to a given maximal distance depending on the election size. We show that, even for a reasonable maximal distance, we still have a high probability of control.

4 Conclusion and perspectives

We have analysed two cases where the control power of the polling institute is very high. We plan to go further and consider a more general model with limited information and competition between two polling institutes.

References


