Cut-Generation Decomposition Algorithms for Uncertain Unit Commitment

Anna Robert[1], Marco Marchese[2], Nouha Dkhili[1], Sami Ghazouani[2], Vincent Leclère[3], Vitor Luiz Pinto de Pina Ferreira[3]

[1] TotalEnergies OneTech R&D Numerical Platform
{anna.robert, nouha.dkhili}@totalenergies.com
[2] TotalEnergies OneTech R&D Hybrid
{marco.marchese, sami.ghazouani}@totalenergies.com
[3] CERMICS
{vincent.leclere, vitor.pinto-de-pina-ferreira}@enpc.fr

Keywords : *unit commitment, stochastic optimization, robust optimization, decomposition algorithms, cut-generation, dynamic programming.*

1 Introduction

The conventional Unit Commitment problem in energy systems management involves optimizing the operation schedule—activation and deactivation—of thermal units to meet the load at minimum cost and/or emissions. Examples of problem formulations can be found in [1]. However, the increased global interest in renewable energy sources, such as solar and wind power, introduces unpredictability in power generation that historically could be reliably dispatched. Uncertain Unit Commitment (UUC) models extend this framework by integrating uncertainties stemming from generation, load, pricing, and/or other factors. These extensions require decomposition-based computational methods to solve these problems efficiently and accurately.

In collaboration with TotalEnergies, our research focuses on an isolated industrial energy-intensive site with known energy load. The energy demand must be met by internal resources: thermal generators, photovoltaic panels, and batteries. The complexity of this problem arises from the interplay of factors like storage and ramping constraints in thermal units, inter-stage coupling, binary variables, and the unpredictable nature of solar production. Ensuring continuous load fulfillment remains imperative for safety and reliability. Yet, an overly cautious approach can result in high energy production costs and emissions.

2 Modeling

2.1 Risk aversion

Modelling uncertain parameters—solar production here—allows a trade-off between robustness and costs. Thus, we need to encode the decision-maker's attitude toward risk into the model. We consider the risk-neutral and risk-averse approaches via the *stochastic* and *robust* settings respectively. In the stochastic setting, we aim at minimizing the expected cost, while in the robust setting, we minimize the cost attained by the worst-case event.

2.2 Information models

The "real problem" requires decisions to be made at each stage before new information (for example, PV production during that stage) is obtained. The resulting issue comes in terms of tractability as the problem quickly grows very large. In particular, the number of binary variables needed to represent the thermal units is exponential with the number of stages at which information is revealed.

In order to resolve this issue, we opted to approximate the information structure of the problem. By separating the information structures behind binary and continuous variables, we can, in turn, decouple them in the optimization problem and use cut-generation algorithms to solve both problems. In our case, we decided that all binary variables are in the first stage—a conservative approximation that nonetheless ensures feasibility. Borrowing from optimal control theory, we call this information structure *Open-loop*, or simply Open. The continuous variables are instead decided to have either progressive knowledge of the uncertainties—which, again borrowing from optimal control nomenclature, we call *Closed-loop*—or with full knowledge—called *Anticipative* in the stochastic optimization literature—, acting as recourse decisions.

These different combinations result in different UUC models, and we highlight the key combinations of interest in Table 1. The Open-Anticipative model could be considered the state-of-the-art in the robust setting. However, as it relaxes the Continuous decisions by using more information and simultaneously overconstraints the Binary decisions by using less information, the model's relationship with the "real problem" is unclear. The compromise we would like to propose is the Open-Closed model, which overconstrains the "real problem".

		Continuous	
		Closed	Anticipative
Binary	Open	Compromise	State-of-the-art [2]
	Closed	"Real Problem"	

TAB. 1: The main different combinations of information models for different types of variables.

3 Algorithms

The stochastic and robust variants of the Open-Closed and Open-Anticipative UUC problems can be solved efficiently with cut-based decomposition methods under certain assumptions on the interdependence of the uncertainty over time (or lack thereof). These algorithms consist of state-of-the-art numerical methods, either applied directly or modified, including both Stochastic and Robust Dual Dynamic Programming algorithms [3, 4]. We examine the performance of the algorithms as well as the solutions obtained from the different models with numerical examples.

References

[1] Miguel F. Anjos and Antonio J. Conejo. "Unit Commitment in Electric Energy Systems". In: *Foundations and Trends® in Electric Energy Systems* 1.4 (2017), pp. 220–310.

[2] Alain Billionnet, Marie-Christine Costa, and Pierre-Louis Poirion. "2-Stage Robust MILP with Continuous Recourse Variables". In: *Discrete Applied Mathematics* 170 (June 19, 2014), pp. 21–32.

[3] Angelos Georghiou, Angelos Tsoukalas, and Wolfram Wiesemann. "Robust Dual Dynamic Programming". In: *Operations Research* 67.3 (May 2019), pp. 813–830.

[4] M. V. F. Pereira and L. M. V. G. Pinto. "Multi-Stage Stochastic Optimization Applied to Energy Planning". In: *Mathematical Programming* 52.1 (May 1, 1991), pp. 359–375.