Pickup and Delivery Problem with Cooperative Robots

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1 Introduction

A new interest emerged among automated vehicles inside warehouse problems, that is cooperative robots. When a package is too large, it requires several robots to cooperate in carrying the package from its pick-up point to the destination. We call this the Pickup and Delivery Problem with Cooperative Robots (PDP-CR). In PDP-CR, we have a fleet of identical robots and a set of tasks. A task has a pick-up point, a destination, and may require multiple robots. We want to complete all the tasks using these starting robots. A task is considered done if all robots needed for the task reach its pick-up point and then go to the destination simultaneously. After the robots complete a task, they may go to other tasks. The goal of PDP-CR is to minimize the makespan, i.e. the time at which the last robot returns to the depot.

The PDP-CR, to the best of our knowledge, is a problem that has not been studied. However, some formulation ideas can be obtained in related problems. The Vehicle Routing Problem (VRP) [5], one of the most important combinatorial optimization problems, focuses on the efficient distribution of goods or services to a given set of customers using a fleet of vehicles normally starting at the central depot. The Pick-up Delivery Problem (PDP) [4] deals with the transportation of goods from an origin to a destination. Synchronization in VRP [1, 2, 3] refers to the constraints that require two or more vehicles to carry out a single task.

The primary goal of our presentation is to introduce the PDP-CR and establish two mathematical models for the PDP-CR. A VRP-based one in which a robot is represented by a vehicle and a flow-based formulation in which a robot is represented as a unit flow. Moreover, we also propose a comparative experimental study between the two models.

2 Problem Description

We have a fleet of I identical robots and a set of tasks J. Every task j is characterized by its pick-up location p_j, its destination d_j, its process time c_j and its number of robots required n_j. The early robots that visit the task must wait for all the robots required to arrive, then they can start to process the task from its pick-up point to its destination.

The problem can be represented on a directed graph $G = (V, E)$. V contains the location of the depot, the pick-up point, and the destination of all tasks. Node 0 represents the depot. All the robots start and end their path at the depot. c_{ij} is the time for any robot to move from task i to task j, that is, the time for any robot to go from d_i to p_j. The goal of the problem is to minimize the makespan, which is the time the last robot returns to the depot.

Instances	Robots	Tasks	VRP Model				Flow Model			
			time (s)	gap	objective	nodes	time (s)	gap	objective	nodes
4	1	5	0.05	0	2056.04	1	0.08	0	2056.04	394
4	1	10	62.17	0	4088.34	975418	543.97	0	4088.34	6308701
4	1	15	7201.39	0.82	6079.04	26056179	7200.69	0.93	6079.04	17657021
4	2	5	0.57	0	2312.82	7	0.55	0	2312.83	565
4	2	10	3490.99	0.15	2951.25	6146442	4368.85	0.38	2963.03	9942544
4	2	15	7204.88	0.82	4870.71	8215701	7200.3	0.91	4769.36	4965917

TAB. 1 – Numerical Results

3 Formulations

Two MILP models will be presented. The first one is based on the VRP by replacing task j that requires n_j robots with n_j tasks that require 1 robot and with synchronization constraints. Another one is based on flow formulations where every robot is represented by a flow unit.

4 Numerical Result

The data to test the formulations are as follows. Number of robots $I \in \{1, 2\}$. Number of tasks $|J| \in \{5, 10, 15\}$. For each pair of chosen (number of tasks, number of robots), we generated 4 random instances according to the following : Robot required for each task $n_j \in \{1, 2, \ldots, I\}, \forall j \in J$. Coordinate of pickup location, destination for each task $(x_j, y_j) \in ([0, 500], [0, 500]), \forall j \in J$. All random generation is done according to the uniform distribution. The distance c_{uv} is the Euclidean distance between node u and node v. The processing time of task j is the distance from its pick-up point to its destination. For each instance, we set the time limit to 120 minutes. The results are reported in Table 1. In the table, every row is an average of 4 randomly generated instances. *time* is the solving time in second, *gap* is the solved gap reported by CPLEX. A smaller gap indicates that the current solution is closer to the optimal solution. If the gap is 0, the instance is solved to optimality. It is calculated as :

$$\text{gap} = 1 - \text{best node/best integer solution}$$

objective is the best objective value found. *nodes* is the number of processed nodes on the branch and bound tree. In our numerical experiments, the VRP-based model typically performs better than the Flow-based model. Both models can solve small instances quickly. When we increase the number of tasks and processed nodes surge dramatically for both models. Hence, improving the formulations or finding good heuristics algorithms could be a good direction.

Références

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