A matheuristic for the 3-stage packing problem

Khadija Hadj Salem[1], Benoit Lardeux[1], Xavier Schepler[2]

[1] L@bISEN, Usine du Futur, ISEN Yncréa Ouest, 33 Quater Avenue du Champ de Manœuvres, 44470, Carquefou, France

{khadija.hadj-salem, benoit.lardeux}@isen-ouest.yncrea.fr

[2] DataLab, Generix Group, 8 Rue Simone IFF, 75012, Paris, France

xschepler@generixgroup.com

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1 Problem definition

This paper considers the three-stage packing problem (3-stage PP), a new Cutting and Packing (C&P) problem variant. According to the typology of [4], the 3-stage PP combines two well-known NP-hard problems : the Bin-Packing Problem and the Pallet Loading Problem. The 3-stage PP consists of 3 phases as follows :

1. **Pallet building stage** : All box types (already containing requested items from different customer orders) are grouped into layers (an arrangement of boxes/box types), and the layers are stacked on the available set of Homogeneous (Hmg : contain boxes of the same type) or heterogeneous (Het : mix of box types from a same customer) pallets. Three building structures are considered : flat, mixed layers, and chimneys.
2. **Pallet stack building stage** : All pallets are therefore stacked in levels in a set of pallet stacks (each one dedicated to a single customer) while respecting some specific practical constraints such as the the fragility, the orientation, etc (see Fig. 1).
3. **Truck loading stage** : All pallet stacks are loaded into a minimum number of trucks of different types and fixed capacity while respecting stability constraints (see Fig. 2).

This problem arises from the analysis of industrial problems encountered by a software company developing solutions for major players in the supply chain field.

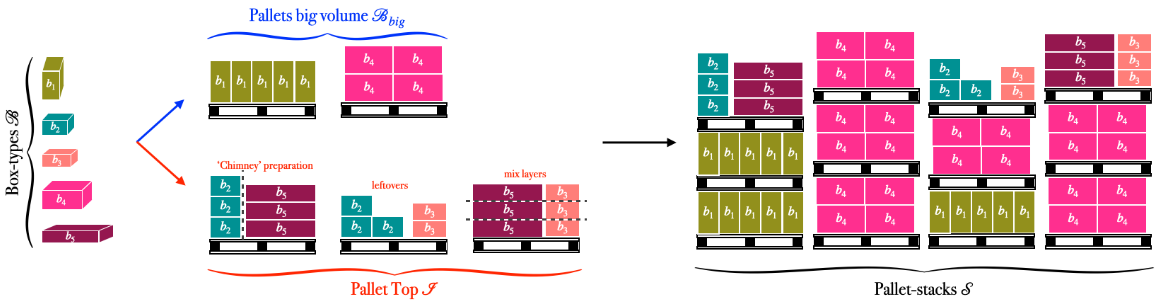

FIG. 1 – Pallets & Pallet stacks building : stages 1 & 2

More formally, let us consider a set of customers $\mathcal{C}$, where each customer c requests one or more orders (set of 3D items). Each order is stored in several boxes from a single box type. A box type b is characterized by its 3D dimensions (width W_b, height H_b, length L_b), a weight Q_b, and a maximum supported weight Q_b^{max}, and a demand d_b (number of occurrences). We also consider a set of pallet types, where each pallet type p has a width W_p, a height H_p, a length L_p, a weight Q_p, and a maximum additional load weight Q_p^{max}. In this variant of the problem, we consider a single type of pallet, as trucks are generally loaded with pallets of the same single type, for example, the standard European Pallet (EPAL : 800 × 1200 mm). The demanded box types are packed on pallets which are stacked. The resulting pallet stacks are then loaded into trucks. We consider a heterogeneous truck fleet. Each truck type $\tau \in \mathcal{T}$ has a 3D load space (W^τ, H^τ, L^τ), and a maximum load weight $Q^{max,\tau}$. A pallet stack loaded in a truck type τ must be placed on one predefined position from the matrix Φ^τ. For example, a semi-trailer truck can load 33 standard EPAL pallet stacks, arranged as shown in Fig. 2.

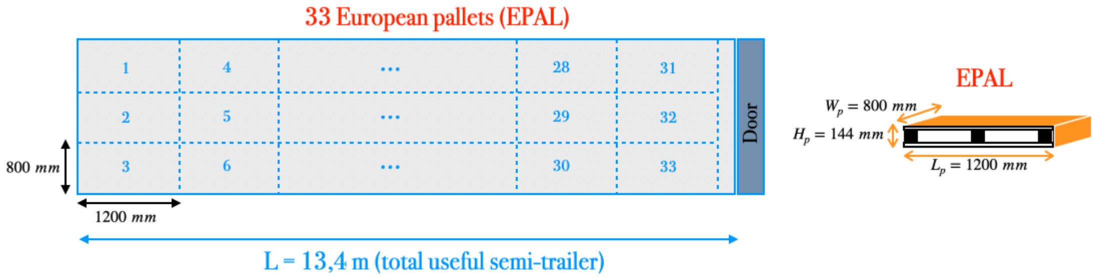

FIG. 2 – Truck loading stage (case of loading semi-trailer by 33 EPALs)

2 Solving approaches

Several studies in the literature on C&P problems have dealt with similar variants. Some of these variants are the two-stage packing problem (loading boxes onto pallets and loading pallets into trucks) that occurs in the distribution process of a Portuguese trading company, tackled by [3]. A two-stage approach (GRASP + tree search procedure) was developed to solve it. We can also cite the multi-container pallet loading problem (building pallets and loading them into trucks) addressed in [1, 2], where several integer linear programming models and a GRASP algorithm have been proposed to explore the specificity of different practical constraints.

In this presentation, to solve the 3-step PP, we first propose a MILP formulation, where three sets of binary/integer/Continuous variables representing the three phases of the problem are considered : loading box types onto Hmg/Het pallets ; building the pallet stacks by arranging pallets ; and loading pallet stacks into trucks.

We then propose a Column Generation-based matheuristic. In the first step, all possible homogeneous pallets $\mathcal{P}^c_{hmg}$, grouping the highly demanded box types into layers, are created. On the other hand, a heuristic is used to generate the set of heterogeneous pallets $\mathcal{P}^c_{het}$ (that can only be placed on the top of a pallet stack) to pack (in mix layers or into chimneys) the leftovers and the set of box types with small demand. Similarly, a heuristic is used to generate an initial set of pallet stacks $\mathcal{S}$ for each truck type $\tau \in \mathcal{T}$ (a combination of Hmg/Het pallets). After that, an adapted version of the classical CG process is applied. It iterates between solving the linear relaxation of the restricted master problem (RMP) and the 1D variable-sized packing problem (1D-VSBPP) pricing sub-problem (subject to height, weight and fragility constraints), by considering a subset of pallet stacks $\mathcal{S}' \subset \mathcal{S}$.

We implemented the proposed approaches and conducted our experiments using randomly generated instances based on assumptions coming from real cases. Numerical results will be given and analyzed during the presentation.

Références

[1] M.T. Alonso, R. Alvarez-Valdés, M. Iori, and F. Parreño. Mathematical models for multi container loading problems with practical constraints. *Computers & Industrial Engineering*, 127 :722–733, 2019.

[2] M.T. Alonso, R. Alvarez-Valdés, and F. Parreño. A grasp algorithm for multi container loading problems with practical constraints. *4OR*, 18 :49–72, 2020.

[3] A. Moura and A. Bortfeldt. A two-stage packing problem procedure. *International Transactions in Operational Research*, 24(1-2) :43–58, 2017.

[4] G. Wäscher, H. Haußner, and H. Schumann. An improved typology of cutting and packing problems. *European Journal of Operational Research*, 183(3) :1109–1130, 2007.