A column generation based heuristic for the integrated vehicle routing and driver scheduling problem

Wenjiao Sun[1], Maxime Ogier[1], Frédéric Semet[1]

Univ. Lille, CNRS, Inria, Centrale Lille, UMR 9189 CRIStAL, F-59000 Lille, France
{wenjiao.sun,maxime.ogier,frederic.semet}@centralelille.fr

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1 Introduction and motivations

Integrated vehicle Routing and driver Scheduling Problems (RSPs) consist in simultaneously planning routes for vehicles and drivers' schedules. Usually, classical routing problems assume that each vehicle is associated with a single driver throughout its entire route. However, when planning over a large horizon, vehicles can be used without interruption over the whole horizon, while drivers have to rest. Hence, in order to better use the vehicles, in this work, we consider that each of them can be driven by different drivers during the planning horizon.

RSPs have been investigated in different transportation applications, including airline industry, railway, trucks, buses [2]. Although these transportation applications have different characteristics, they all share several main similarities. We denote by task a trip from two locations with predertimned departure and arrival times. A task has to be performed by one or several combined vehicles, and one driver has to be driving. The combined vehicle case typically appears for trains and trucks with trailers. Note that drivers can also be passengers on the vehicle. Given a set of tasks, the aim is to plan the sequence of tasks to be performed by each vehicle and each driver such that each task is performed by the required number of vehicles and at least one driver. Our objective is to propose a generic model for these different applications, and develop an efficient algorithm to solve large-size RSPs.

Similar works on RSPs not dedicated to a specific application have been proposed by [1]. However, the problem is restricted to two depots, assumes that drivers can have very long working times, and considers that only a single vehicle performs a specific task.

2 Problem definition and a generic model

Three main entities are considered : vehicles, drivers, and tasks. At the beginning of the planning horizon, drivers and vehicles start from any depot. A task is defined by its origin and destination locations, and by its departure and arrival times. Two tasks T_A and T_B can be performed consecutively ; that is, there is a possible connection from T_A to T_B only if : (i) the destination of T_A is the origin of T_B, and (ii) the departure time of T_B is after the arrival time of T_A. The difference between the departure time of T_B and the arrival time of T_A is named the connection time.

Specifically, given a fleet of vehicles, a set of drivers with their departure depots, and a set of tasks, the aim of the RSP is to provide a sequence of tasks for each vehicle and each driver such that each sequence is composed of possible connections and all tasks are performed by a required number of vehicles and by at least one driver. Each driver has to end at the same depot he started at. During the planning horizon, a driver may change vehicles, rest at certain locations, or be a passenger in some tasks. When the connection time between two consecutive tasks is too short, drivers have to stay in the same vehicle and keep the same working status.

In addition, drivers must respect the regulations regarding limited driving hours and working time. It is not required that a vehicle ends at its starting depot, but at the end of the planning horizon, each depot has to contain as many vehicles as at the beginning of the horizon.

The objective is to minimize the total cost which consists of three parts : (i) the cost of vehicles, (ii) the drivers' salary, and (iii) the penalty when drivers change vehicle or working status (from driving to passenger and reciprocally).

3 Solving method

We first propose a compact mixed-integer programming formulation for the integrated RSP. Using a commercial solver, we can only solve small-size instances with the compact formulation. To handle large-size instances in a reasonable amount of time, we decompose the integrated RSP into two subproblems (1) a driver scheduling problem, and (2) a vehicle routing problem. We then solve the RSP with a two phases heuristic : first, we solve the driver scheduling sub-problem, and then, knowing the driver schedules, we solve the vehicle routing sub-problem.

The first sub-problem is challenging to solve with a compact formulation, so we propose an extended formulation. To handle the exponential number of variables, we developed a column generation based heuristic with performance guarantee. A lower bound is obtained by solving the linear relaxation of the extended formulation with column generation, and then an upper bound is obtained by solving a MIP that corresponds to the extended formulation restricted to the columns generated to provide the lower bound. In column generation, the pricing problem is solved by means of a dedicated labeling algorithm with dominance rules. If the solving of the linear relaxation does not end before the preset time limit, we use a Lagrangian bound to report a valid lower bound.

The second sub-problem, which is the vehicle routing problem, takes as input the drivers' schedules solution of the first sub-problem. When solving the vehicle routing sub-problem, we allow some flexibility in the driver scheduling since we also decide for each driver which trips he is driving. while ensuring the maximum driving duration is respected and each task has one driving driver. The second sub-problem is formulated as a MIP.

4 Conclusion and perspectives

A general model for the RSP has been proposed. The model covers the case of drivers sharing vehicles and changing their working status, as well as combined vehicles performing the same task during the planning horizon. We generated 42 random instances of small, medium, and large sizes, with up to 300 trips. In the decomposition approach, the driver scheduling sub-problem is solved in a few seconds for small-size instances and a maximum of 30 minutes for large-size instances. The optimality gaps are of good quality, less than 4%. The vehicle routing sub-problem is solved very quickly, in less than 7 minutes. We evaluated the quality of the solution with respect to optimal solutions for small-size instances. The two-phases approach provides very good results in general, except for some instances.

For future works, we can build an iterative approach based on the proposed two-phases heuristic. Moreover, we would like to propose a more generic integrated RSP in which tasks have no predetermined time. Thus, the starting time of the tasks becomes a decision.

Références

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