## Optimization Models for Robust Telecommunication Network Design and Efficient Resource Allocation

John J. Quiroga-Orozco<sup>1,2</sup>, Boris Detienne<sup>2</sup>, Eric Gourdin<sup>1</sup>, Nancy Perrot<sup>1</sup>, Pierre Pesneau<sup>2</sup>

 <sup>1</sup> Orange Innovation, Orange Gardens, F-92320 Châtillon, France {johnjairo.quirogaorozco, eric.gourdin, nancy.perrot}@orange.com
<sup>2</sup> Univ. Bordeaux, CNRS, INRIA, Bordeaux INP, IMB, UMR 5251, F-33400 Talence, France {boris.detienne, pierre.pesneau}@math.u-bordeaux.fr

**Keywords** : robust optimization, two-stage problem, k-adaptability, network design.

## 1 Introduction

Telecommunication networks rely more and more on virtualization and softwarization technologies. Thanks to virtualization the network functions can be installed and processed on the network nodes only when and where needed, depending on the communication services that are planned, and the corresponding traffic flows can be transported on the networks according to more efficient routing algorithms that can be updated dynamically. These technologies bring huge flexibility in the ways of managing and controlling both the network elements and the traffic data that is transported within the network.

Designing the networks usually follows three phases depending on the decision levels: strategical (long term), tactical (medium term), and operational (short term). Long-term decisions consist of establishing the network topology: the nodes, links, and capacities on links and nodes. In the medium term, a set of *Virtual Network Functions* (VNFs) has to be installed in the network at low cost. In the short term, for each commodity service request, a latencyconstrained routing path has to be found where the data must be processed by VNFs following a pre-established order. The pre-established VNFs order requested by each commodity is called *Service Function Chain* (SFC).

The Virtual Network Function Placement and Routing Problem (VNFPRP) aims to define on which nodes of the networks VNFs should be placed and to route the demands on the network following their SFC. This problem is a very challenging combinatorial optimization problem [1, 5]. It becomes even more difficult when the amount of bandwidth requested by each commodity presents uncertainties.

In this work, we consider the VNFPRP with uncertain demands. We propose various models using the robust optimization paradigm where the uncertainty on the demands is represented by a bounded polyhedron built on hose constraints [2].

## 2 Models for the VNFPRP with uncertain demand

In [5], the authors propose a mixed integer programming formulation for the deterministic VNFPRP, where the demands are known in advance. One drawback of this formulation is that it does not allow non-elementary paths (paths admitting sub-circuits) to route a demand from its origin to its destination. However, depending on the sequence of VNFs to be applied on commodities, routes that visit the same node several times for different VNFs can improve the quality of the solutions, in particular in sparse networks.

Based on [4], we propose two deterministic models for VNFPRP allowing non-elementary paths. The first one, called the multi-layer model, consists of creating as many copies of the network (called layers) as there are virtual functions. A given commodity will follow a path from the source node on the first layer to its sink node in the last layer, jumping to the next layer each time it applies a VNF. The second model, called multi-hop, decomposes the path of a commodity into segments (hops) delimited by the VNFs and represents each segment by a set of variables.

These two deterministic models are used as base formulations to address uncertainty on the demands. Our first approach uses the classical duality-based reformulation [3] to derive deterministic MILP models of their static robust counterparts, which assume that all decisions are *here-and-now* (*i.e.* have to be taken before the actual data is revealed). Our second approach allows postponing operational decisions until the uncertainty is revealed. To cope with the integrality restrictions on the *wait-and-see* variables, we use the *k*-adaptability paradigm [6]. In the first stage, capacities that should be reserved for each VNF on each node are determined, as well as *k* alternative routing solutions for the commodities. Once the uncertainty is revealed, the best of the *k* solutions according to those demands will be implemented to route them.

We present numerical experiments assessing the benefits of considering wait-and-see routing decisions over static routing.

## References

- Zaid Allybokus, Nancy Perrot, Jérémie Leguay, Lorenzo Maggi, and Eric Gourdin. Virtual function placement for service chaining with partial orders and anti-affinity rules. *Networks*, 71(2):97–106, 2018.
- [2] Walid Ben-Ameur and Hervé Kerivin. Routing of uncertain traffic demands. Optimization and Engineering, 6(3):283–313, Sep 2005.
- [3] A. Ben-Tal and A. Nemirovski. Robust solutions of uncertain linear programs. Operations Research Letters, 25(1):1–13, August 1999.
- [4] Thales Fernandes de Castro. Instanciation optimale et routage des chaînes de service dans les réseaux software defined networking, 2016. Rapport de stage de fin d'études d'ingénieur 3ème année / Master 2(M2AD).
- [5] Ahlam Mouaci, Éric Gourdin, Ivana LjubiĆ, and Nancy Perrot. Virtual network functions placement and routing problem: Path formulation. In 2020 IFIP Networking Conference (Networking), pages 55–63, 2020.
- [6] Anirudh Subramanyam, Chrysanthos E. Gounaris, and Wolfram Wiesemann. Kadaptability in two-stage mixed-integer robust optimization. *Mathematical Programming Computation*, 12(2):193–224, Jun 2020.